Duodecimal

Arithmetick:

VIZ.

Notation.	Reduction.	
Addition.	Extraction of the —	Square, and Cube Roots.
Subtraction.		
Multiplication.	Rule of proportion —	Direct, and Reverse.
Division.		

Duodecimally performed, and very Practically applied to the measuring of all sorts of Superficies, and Solids, as *Board*, *Glass*, &c. *Timber*, *Stone*, &c.

Robert Midgeley.

But Chiefly to the

Ganging of all sorts of Brewers Tuns and Casks, To find the whole Content, or the Vacuity or Remaining Liquor of either, and that with more Ease and Expedition, than by Vulgar or Decimal Arithmetick. Very Useful for all sorts of Men, aswel Gentlemen as others, but especially for *Merchants*, *Writing Masters*, and all Measuring *Artificers*. And all the Rules made Plain, and Easie for the meanest Capacity.

By *JOSHUA JORDAINE* of *Exon*, Philo-Accomptant.

LONDON, Printed by *John Richardson* for the Author, and are to be Sold by *John Taylor* at the Ship in Pauls *Church Yard*. 1687.

Licensed,

March the 8th. 1687.

Robert Midgeley.

TO THE

KING'S

Most Excellent

MAJESTY

JAMES II.

(By the Grace of GOD)

Of England, Scotland, France & Ireland,

Defender of the Faith, &c.

May it please Your Majesty,

HE that [illegible]ly consider, that [illegible]s are the Founda[illegible] our English Measures, will fo[illegible]e that nothing can be more natural and genuine for the

TO THE
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Of *England*, *Scotland*, *France* & *Ireland*,

Defender of the Faith, &c.

May it please Your Majesty,

HE that shall but duly consider, that Duodecimals are the Foundation of all our English Measures, will soon see that nothing can be more natural and genuine for the

the finding of Superficial and Solid Contents, than a *Duodecimal Arithmetick*, which consideration sway'd me to the Composing of One; and when I had compleated it, I could not think how better to Apply it than for Your Majesties Service, (the First Fruits being Your Majesties due,) and therefore applyed it to the Mysterious *Art of Gauging*, and to all other Superficies and Solids as preparatory thereunto: Not doubting, but that by the Plainness, Easiness, Expedition and Certainty of the way, beyond any other yet used, so many will be encouraged to the Study thereof, as the number of able Artists, will be much increased thereby, to the great advancement of the said Art. And

my

my aim being only levell'd at such an Effect, I now make bold in all Humility to present it to Your Majesty, to let Your Majesty see the Zeal that I have to throw what I can (if but a Mite) into Your Majesties Treasury; which being my All at present, I Humbly implore, as Your Majesties Royal Pardon for this my presumption, so Your Gracious Acceptance of these my Endeavours, which if Your Majesty please to Grant, it shall be a sufficient Incouragement, not onely to a further Improvement of this, but to the compleating of some other things, which I have in pursuit for the good of the Publick. In the mean time, that God would bless Your Majesty with a

Long and Prosperous Reign on Earth, and after that with a Crown of Immortal Glory in Heaven, is, and ever shall be, the Hearty Prayer, of,

Your Majesties most Loyal

and Obedient Subject,

Joshua Jordaine.

To the Right Worshipful

Sir *Denny Ashburnham*, Bar.
Sir *John Friend*, Kt.
Francis Parry, Esq;
Charles Davenant, Doctor of Laws.
Felix Calverd,
Nath. Horneby, } Esquires.
Rich. Graham,

Chief Commissioners and Governors for the Management and Receipt of His Majesties *Revenue of Excise*, and *Hearth-money*, within the Kingdom of *England*, &c.

May it please your Worships,

THE Learned and Able Mathematician Sir Jonas Moore, *for some time since wished for a composure of a* Duodecimal Arithmetick; *and I now present Your Worships with one Compleated*

pleated, having been much sollicited by divers Mathematicians and other Artists for its Publication. And I thought my self obliged in Duty to His Majesty (the First-fruits being his due) to apply it in this first Edition, *to that whereby it might be most serviceable to him, and therefore have appli'd it to the Mysterious Art of* Gauging, *and have also appli'd it to the measuring of all other Superficies and Solids, as preparatory thereunto: And have Reduced the whole Art to that onely Rule, whereby Merchants do so Expedite their comptings,* viz. *The Rule of* Practice: *And have made a Segment Line applicable to all sorts of Casks, for the finding the vacuity or remaining Liquor of either as long as the Liquor cuts the heads, and if time had permitted, had made one whereby to have*

found

found it whether the Liquor cuts the heads or any part of the sides, but I intend (God giving Life and Health) in a short time to do it, a thing never yet done by any, though much endeavour'd for by many; and shall also demonstrate the reason thereof, and of several other things very useful to Gaugers : *So that doubtless the plainness, easiness, brevity, & certainty of this way of* Gauging *beyond any other yet used, will now incourage the more by many to the Study thereof, to the great advancement of the said* Art *in the increase of able* Artists, *especially (your Worships being so highly concerned in, and so well qualified for this Affair) if* R*ecommended to the* Gaugers *by you, and therefore that you would so* R*ecommend it, is my humble* Request, *as knowing that such a* R*ecommendation*

will

will give the Book so free reception and so ready an entertainment, as will be not onely a good [illegible] *Recompence for my labour in* Composing, *and my charge in* Printing, *but also a good* Encouragement *to a further* Improvement; *not doubting but that as my zeal for the promoting this his Majesties* Concern *hath carryed me thorow the work, so the great zeal which your VVorships have for the same, will not only move you to pardon my* presumption, *but also kindly to accept of these my* Endeavours.

Now, that God would Bless Your VVorships in all Your undertakings, both for His Majesty and Your Selves, and with Increase of Honour and Felicity, is the Hearty Prayer of

Your Worships

Most Humble Servant,

JOSHUA JORDAINE.

TO THE

READER,

Courteous Reader,

I Present thee with a *Duodecimal Arithmetick*, long wisht for by many, and which also, by many I have been much sollicited for its publication, urging me with the great Usefulness thereof to all sorts of men, especially *Merchants*, and all sorts of Measuring *Artificers*, as being most practical, and most genuine for their several purposes. But what need I to make any Apology for the Book, when the very Title is a sufficient one of its self; every other Trading-man as well as the Merchant, knowing, that there can be no shorter nor easier method for reckoning, Casting, or Compting, than by Duodecimalls, *viz.* the Aliquot parts of 12; And therefore I shall no further commend it, than

to

to thy ſtudy and practice, and when thou underſtand'ſt it, I ſhall not doubt of thy own commendation. And for thy incouragement (if unlearned) I have made uſe of no other terms, but what the meaneſt capacity may underſtand, (my intention being not to amuſe, but inſtruct,) and therefore have made the Rules ſo plain, and eaſie, as that thou maiſt ſoon attain them without a Tutor, (provided thou underſtand'ſt but Multiplication and Diviſion in Common Arithmetick:) and have as plainly applyed it to the meaſuring of all ſorts of Superficies and Solids, and amongſt them, and chiefly, to the Myſterious *Art of Gauging*.

As to the Merchant, *The Rule of Practice*, is well known to be (as I may ſay) his right hand, and that Rule by this Arithmetick (I think I may preſume to ſay) is now improved to that height, that that hand will be by far the more dext'rous, for by this Arithmetick, all the trouble of his Reductions, and abbreviations is ſaved,(there being in this no need of either) as he may plainly ſee by the taſte I have given him in the *Poſtſcript*.

As to Meaſuring Artificers, whether of *Board*, *Timber*, *Stone*, *Wainſcot*, *Painting*, &c. I have ſo contrived, that whether you meaſure by the foot or the yard ſquare, your work will be ſtill the ſame, *viz.* Nothing but Multiplication in Duodecimalls, which is no other than the Merchants ſaid *Rule of Practice*.

As to Ship-Wrights, Gunners, and all others to whom the Square and Cube Roots are useful, the Extracting of those Roots Duodecimally, is not only easier and plainer, but the valuation of the Fractions sooner found, and nearer to the Truth, than by any other way whatsoever.

As to Gaugers, because of the intricacy of your work, I have (though with much pains and trouble) reduced the whole mystery of your Art, to that one and the same General Rule also, viz. *The Merchants Rule of Practice*, and so not only made it easie for your Understandings, but for your Purses too; having so contrived all your Measures for Brewers Tuns, as that you may carry them in a Box portable enough for the Pocket, and so the charge will not be much, though I have given direction for the carrying of those Measures in an underhand Cane, but that will be more chargeable.

As for Cask Gauging, I have made a Circle Gauge (as I call it) both for Ale and Wine, which onely lain on the Diameter gives the whole Content of the Circle, or the 2 thirds or 1 third thereof as you may have occasion, (provided the diameter exceeds not 6 foot.) I have also made a Segment-Line to find the vacuity or remaining Liquor of Casks, very proper for a Cylinder, and by the direction given may be as proper for a Spheroidal, Parabolical and Conical Cask also, as long as the Liquor cuts the heads. Though if some hindrances had not happened, I had made one to have answered whether the Liquor cuts the heads, or any part of the sides, but if

it ſhall pleaſe God to give me Life and Health, I hope in a little time to perform it, and not only ſo, but clearly to Demonſtrate the reaſon thereof, and of ſeveral other things which now lyes hid from the Gauger.

As for the Circle-Gauge, three foot of it may be carried in an underhand Cane, and may be ſo made to draw, as that the Cane it ſelf may ſerve for the other 3 foot, and ſo not very chargeable neither.

And if any fancy to continue the uſe of the Semi-Circle, if it be but Duodecimally divided, he may uſe it with this Arithmetick. And alſo if any fancy to work by the Line of Numbers, I have given direction to Mr. *Iſaac Carver* of *Redriff*, for the making a Duodecimal One. And whereas ſome of you underſtand Decimal Arithmetick very well, and may not have time to ſtudy the Duodecimal, yet having this Book, if you have all the meaſures Decimally divided, you may work Decimally (obſerving the ſame Rules) throughout, though the Duodecimal would be much eaſier and ſpeedier, and leſs ſubject to miſtakes had you time to attain it.

Laſtly, As for the Writing Maſters of this Nation, as it would be too long, ſo I hope it would be as needleſs to inform you, either how uſeful this Arithmetick will be to you, or how to apply it in your inſtructing and fitting youth for their ſeveral Trades: How ever give me leave to tell you, that it will not onely be uſeful in inſtructing them for meaſuring Trades and Imployments, but for Mer-

chan-

chandize, and amongst many other things (to which the applying of it I leave to your own discretion,) it will be very advantagious in all sorts of Exchanges, as you may see by the Specimen in the *Postscript.*

All which premises considered, I hope you will grant it a kind Acceptance, and a Friendly resentment, it being intended for a Publick good, and Composed according to the Ability which God hath given me. So wishing you good success in your Studies, I remain A Lover of Art, and a True Friend to all that Love it,

Joshua Jordaine.

THese are to Certifie all Gentlemen, Artists and others, whom it may Concern, That at the Authors Request, I have perused that Ingenious Treatise of *Duodecimal Arithmetick*, together with all its Applications, and do conceive it is not onely useful in all the Parts of Vulgar Arithmetick, but of most Excellent Use in the Measuring of all sorts of Superficies and Solids, and chiefly in *Gauging*, wherein I conceive it far surpasseth any way that I have seen ever invented, both for brevity and ease, having reduced the whole Art to one general Rule, *viz. The Common Rule of Practice.*

Tho. Baker.

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BOOK. I.

Of Duodecimal Arithmetick.

10. To

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Of Superficies of Solid Bodies.

II. Of

II. Of Solids.

4. *Hav-*

BOOK. III.

Duodecimall Gauging.

1. *Bound-*

The Contents.

1. *Bound-*

Duo-

Duodecimal Cask-Gauging.

ALL the Inſtruments relating to this Book, both for Menſuration and Gauging, are Made by Mr. *Iſaac Carver*, Mathematical Inſtrument Maker, at the Globe-Sun-Dial on *Horſly down* near *Redriff*, *London*, Who alſo maketh Inſtruments for any other part of the Mathematicks in Silver, Braſs, Ivory or Wood.

IF any pleaſe they may be Inſtructed either in the Arithmetical, Meaſuring or Gauging part, or in the whole of this Book by the Authors Kinſman, Mr. *John Jordaine* of *London*; to whom alone he hath Taught the ſame, who being at preſent unſettled, in a ſhort time will give Notice where he may be found.

The

The *ERRATA.*

P*Age* 96. *line* 9. *read* 17 $\frac{1}{2}$. *p.* 205. *l.* 28. *r. and one prism.* *p.* 470. *l.* 27. *r.* 4 $\frac{1}{2}$.

And whereas, if the Multiplicand, and the Multiplyer have both Duodecimals, they may be multiplied Duodecimally though they differ never so much in kind, the question in *Page* 275 may be thus wrought (being an oversight there,) *viz.*

barr.	′	″		s.	d.	
889	4	8	at	2	9	per barrel.
2	9					
1778	9	4				
444	8	4				
222	4	2				
24415	9	10				

Facit 122 — 5 — 9 — $\frac{3}{4}$ and $\frac{1}{12}$ of a penny.

Duodecimal Arithmetick:

BOOK. I.

The INTRODUCTION.

FOR the Learners better underſtanding of the enſuing work, (ſuppoſing him to be able to number, add, ſubduct, and divide, according to common *Arithmetick*,) he muſt know that there are Three ſorts of *Quantity*.

Viz. { *A Line*, which hath length, but no breadth.
A Superficies, which hathlength, &breadth, but no depth.
A Solid, which hath length, breadth, and depth. }

And that each of theſe kinds of Quantity is commenſurable by ſome common meaſure thereunto aſſigned, as a Line, by a Line of inches, feet, poles, furlongs, &c. And a Superfiices by a Superficies, as the ſquare inch, ſquare foot, ſquare yard, ſquare perch, &c. And a Solid by a Solid, as the cubick foot, &c. So when it is found how many inches, feet, poles, furlongs, &c. are contained in any Line, the length of that

Line is ſaid to be known. And when it is found how many ſquare inches, ſquare feet, ſquare yards, or ſquare perches, are contained within any ſuperficies, the Content, or Area of that ſuperficies is ſaid to be known. And alſo when it is found how many cubick inches, cubick feet, &c. are contained in any Solid, the Content of that Solid is ſaid to be known.

Then he muſt obſerve that a ſuperficial foot * ſquare, is divided in its length into 12 parts, which may be called long inches, or inches of the ſquare foot, or rather Primes, 12 of which make the ſquare foot. Then, that 'tis divided again in its breadth into 12 other parts, cutting each inch into 12 parts alſo, which may be called parts of the long inch, or of the inch of the ſquare foot, or rather Seconds, 12 of which make the ſaid inch, every ſuch part being one inch ſquare, and 144 of thoſe parts do make a foot ſquare, (as may plainly appear by the following figure,) and each of thoſe parts may be divided into 12 other parts and be called thirds, and ſo *ad infinitum.* And accordingly, the Carpenters Rule, (or any other Rule you intend to make uſe of, for this *Arithmetick*,) muſt be Duodecimally divided; for upon this Baſis is the whole enſuing work founded, and therefore this figure may be called.

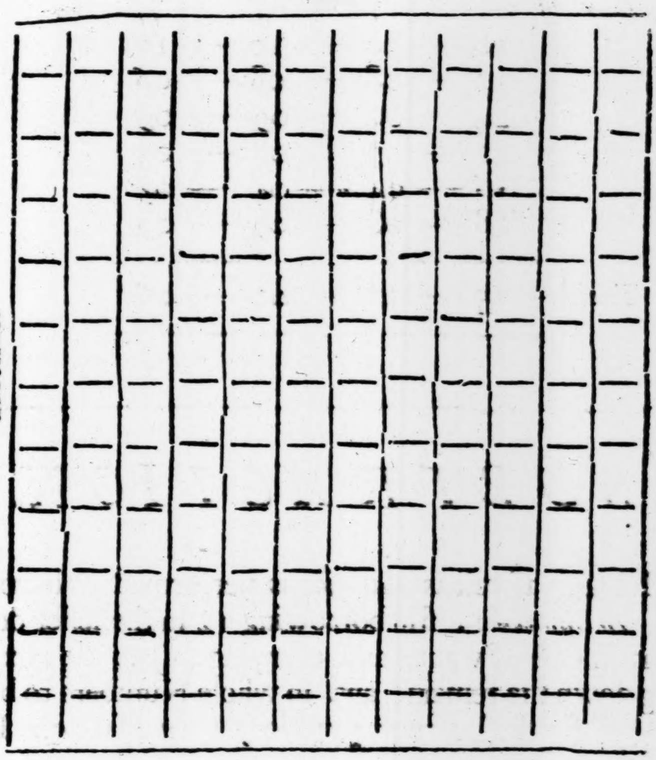
* *Any other ſquare meaſure may be divided in the like manner.*

NOTA-

NOTATION.

Primes.

Seconds.



ADDITION.

THis is performed as in common *Arithmetick*, begining firſt at the leaſt denomination, making a point or prick at every 12 in every row of the leſſer denominations, ſtill carrying as many points or pricks as you make, to the next row ſetting down the odd, and at Integers or whole numbers to carry one for every 10. yet though it be ſo plain I have ſate down one Example for the Learners practice,

which ſuppoſing it to be in foot meaſure, or what you pleaſe, you may call it Integers, Primes, Seconds, Thirds, &c.

Example.

		′	″	‴
	10	10	10.	10.
	12	08·	08·	06
	24	06·	09·	09·
	32	07	07	05
	16	04	04	03·
	18	08·	09·	09
Facit	115	11	02	06

SUBTRACTION.

THis alſo aſwell as *Addition* is performed as in common *Arithmetick*, ſtill borrowing 12 in the leſſer denominations in caſe of want, and carrying (1) to the next row, ſetting down the remain, and in whole Numbers to borrow 10. And for the Learners practice, I have ſate one Example of this alſo.

Example.

		′	″	‴
From	38	08	06	04
Take	29	09	09	08
Remain	08	10	08	08
Proof	38	08	06	04

MVLTI-

MULTIPLICATION.

IN this Rule lyes the greateſt Myſtery, and Excellency of what is hereby intended; and therefore here I ſhall lay down all ſuch Rules as are neceſſary for the attaining thereof, and ſhall make them ſo plain, as the meaneſt capacity may apprehend them. But before you proceed to the Rules you muſt be thorow perfect of theſe four enſuing, eaſie and common Tables.

Viz. Firſt.

4 Times	4	is	16	7 Times	7	is	49
	5		20		8		56
	6		24		9		63
	7		28	8 Times	8	is	64
	8		32		9		72
	9		36	9 Times	9	is	81
5 Times	5	is	25	10 Times	10	is	100
	6		30				
	7		35				
	8		40				
	9		45				
6 Times	6	is	36				
	7		42				
	8		48				
	9		54				

Second.

2	Times 12 is	24
3		36
4		48
5		60
6		72
7		84
8		96
9		108
10		120
11		132
12		144

Third.

2	Times 11 is	22
3		33
4		44
5		55
6		66
7		77
8		88
9		99
10		110
11		121
12		132

Fourth.

6	is the	$\frac{1}{2}$	Of 12.
4		$\frac{1}{3}$	
3		$\frac{1}{4}$	
2		$\frac{1}{6}$	
$1\frac{1}{2}$		$\frac{1}{8}$	
1		$\frac{1}{12}$	

Then having learnt the Three first Tables, and acquainted your self well with the parts of the last, you must learn the use and practise of those parts; for which purpose you must diligently observe, and perfectly learn these following Rules, *viz.*

Rule 1.

To multiply any given number consisting of Integers, Primes, Seconds, Thirds, &c. by any number of Primes that are an Aliquot part of 12.

You are to take the Aliquot parts of 12 which you find by the Fourth Table those Primes by which you are to multiply, do make.

1. *Ex-*

1. *Example.*

6 Primes is the $\frac{1}{2}$. Multiply 14 —— 08′ —— 10″ By 6 Primes.

7 —— 04 —— 05 *Facit.*

2. *Example.*

4 Primes is the $\frac{1}{3}$ Multiply 10 – 10′ – 08″ By 4 Primes.

3 – 07 – 06 — 08 *Facit.*

3. *Example.*

3 Primes is the $\frac{1}{4}$ Multiply 18 – 08′ – 06″ By 3 Primes.

4 – 08 – 01 — 06 *Facit.*

4. *Example.*

2 Primes is the $\frac{1}{6}$ Multiply 16 – 04′ – 09″ By 2 Primes.

2 – 08 – 09 — 06 *Facit.*

5. *Example.*

1 $\frac{1}{2}$ prime is the $\frac{1}{8}$ Multiply 14 – 10′ – 10″ By 1 pr. & 6 seconds.

1 – 10 – 04 — 03 *Facit.*

Now here are two things to be noted.

Firſt, That what remains in taking your parts, whether it be at Integers, primes, ſeconds, Thirds, &c. you muſt account it ſo many of thoſe parts as you take, at every place reſpectively; as if it be at integers, you muſt account it ſo many parts of the integer; if at primes, ſo many parts of a prime, &c. As in the ſecond Example in taking the $\frac{1}{3}$ for 4 primes, there remained at integers 1, which muſt be accounted $\frac{1}{3}$ of the integer which is 4 primes to be carried to the place of primes; then at primes there remains 1 again, which muſt be accounted $\frac{1}{3}$ of a prime, which is 4 ſeconds, to be carried to the place of ſeconds; and at ſeconds there remained 2, which muſt be accounted $\frac{2}{3}$ of one ſecond, which is 8 thirds, to be carried to the place of thirds, &c.

Secondly, That every anſwer is given in ſquare integers, and primes, ſeconds, thirds, &c. of the ſquare integer.

Rule 2.

To multiply by any number of primes under 12, which are not Aliquot parts of 12, as by 5. 7. 8. 9. 10. or 11. primes.

This may be performed Two ways.

Firſt, you may divide them into Aliquot parts, and work as before, as if you were to multiply by 5 primes, ſay 4 primes is $\frac{1}{3}$, and 1 prime is $\frac{1}{12}$, or you may ſay that 1 prime is the $\frac{1}{4}$ of the $\frac{1}{3}$ or 4 primes. So for 7 primes, ſay 6 is the $\frac{1}{2}$, and 1 is the $\frac{1}{12}$, or 1 is the $\frac{1}{6}$ of $\frac{1}{2}$, ſo for 9 primes, ſay 6 is the $\frac{1}{2}$, and 3 is the $\frac{1}{4}$, or 3 is the $\frac{1}{2}$ of $\frac{1}{2}$, &c.

1. *Ex-*

1. *Example.*

Multiply 10 —— 10′ —— 07″ By 5 Primes

3 —— 07 —— 06 —— 04
10 —— 10 —— 07

4 —— 06 —— 04 —— 11 *Facit.*

2. *Example.*

Multiply 10 —— 10′ —— 07″ By 7 Primes.

5 —— 05 —— 03 —— 06
0 —— 10 —— 10 —— 07

6 —— 04 —— 02 —— 01 *Facit.*

3. *Example.*

Multiply 11 —— 11′ —— 11″ By 9 Primes.

5 —— 11 —— 11 —— 06
2 —— 11 —— 11 —— 09

8 —— 11 —— 11 —— 03 *Facit.*

4. *Example.*

4. Example.

Multiply 14 —— 08′ —— 10″ By 11 primes.

4 —— 10· —— 11· —— 04·
4 —— 10· —— 11· —— 04
3 —— 08 —— 02 —— 06

13 —— 06 —— 01 —— 02 *Facit.*

In this last Example I took $\frac{1}{3}$, and sate it down twice for 8 primes, and then I took the $\frac{1}{4}$ for 3 primes.

Or secondly, you may place a cypher in stead of an integer under the last place, and the primes one place farther back to the right hand, and then multiply by the primes as if they were integers, or whole numbers, carrying 1 for every 12, and setting down the odd, setting the first figure under the figure you multiply by. And here the two first Examples will be sufficient for demonstration.

1. Example.

Multiply 10 —— 10′ —— 07″ By 5 primes.
0 —— 5

4 —— 06 —— 04 —— 11 . *Facit.*

2. Example.

Multiply 10 —— 10′ —— 07″ By 7 primes.
0 —— 07

6 —— 04 —— 02 —— 01 .

Rule 3.

Rule 3.

To multiply any given number in integers, primes, ſeconds, thirds, &c. or only in primes, ſeconds, thirds, &c. by any number of ſeconds, or thirds, &c.

Multiply according to former direction for primes, placing the product continually one place back for ſeconds, two places back for thirds, &c. that is to ſay, back towards the right hand. Or you may multiply according to the ſecond direction of the former rule, placing a cypher under the laſt place inſtead of an integer, and alſo another cypher in the place of primes, and then the ſeconds and thirds, &c. following, to the right hand, and multiply as before directed.

Examples of the firſt way.

1. *Example.*

Multiply 8 — 04′ — 06″ By 6 ſeconds.

. — 4 — 02 — 03 *Facit.*

2. *Example.*

Multiply 10 — 07′ — 08″ By 4 ſeconds.

03 — 06 — 06 — 08 *Facit.*

3. *Example.*

primes ſeconds

Multiply 00 — 6 — 08 By 6 ſeconds.

03 — 04 . *Facit.*

4. Ex-

4. *Example.*

Multiply 12 — 10′ — 10″ By 8 Thirds.

04-03-07-04·
04-03-07-04·

Facit — · — 08-07-02-08·

The Anſwer to this laſt Example is 8 ſeconds, 7 thirds, 2 fourths, and 8 fifths, which if it were foot meaſure, muſt be accounted 8 parts of the long inch, 7 parts of one of them, 2 parts of one of thoſe, and 8 parts of one of them.

Examples of the ſecond way.

1. *Example.*

Multiply 8 — 4′ — 06″ By 6 ſeconds.
· 0 — 0 — 6

4 — 2 — 3 · — 0 *Facit* 4 prim. 2 ſec. (3 thirds

2. *Example.*

Multiply 12 — 10 — 10· By 8 ſeconds and 5 Thirds.
0 — 0 — 8 — 5

5 — 4 — 6 — 2
8 — 7 — 2 — 8

· — 9 — -00 — 7 — -02 — -02 *Facit.*

Rule 4.

Rule 4.

To multiply any number of primes, and ſeconds, by 1 prime, or 1 ſecond, &c.

Place the given ſumme 1 place back to the right hand for primes, two places back for ſeconds, and three for thirds, &c.

1. *Example.*

Multiply 00 — $10^{\prime}$ — $10^{\prime\prime}$ — By 1 prime.

Facit — 10 — 10, *viz.* 10 ſec. 10 thirds.

2. *Example.*

Multiply 00 — $11^{\prime}$ — $11^{\prime\prime}$ — $11^{\prime\prime\prime}$ — By 1 prime 1 ſec.

11 — 11 — 11
11 — 11 — 11

01 — 00 — 11 — 10 — 11 *Facit.*

Rule 5.

To multiply any number of integers, primes, ſeconds, &c. by 1 prime, or 1 ſecond, &c.

Firſt, for 1 prime, take the $\frac{1}{12}$ of the integers, and if it amounts to 1 integer, or more, ſet it under integers, and the primes remaining (if any) under primes, and then for the primes

primes and ſeconds given, place them one place back to the right hand.

Secondly, For 1 ſecond, take the $\frac{1}{12}$ of the integers and count it ſo many primes, and the remain ſo many ſeconds, and place the primes and ſeconds given, two places back to the right hand, &c.

Example of the firſt.

Multiply 15 — 10′ — 07″ By 1 prime.

1 — 03 — 10 — 07· *Facit.*

Example of the ſecond.

Multiply 16 — 08′ — 06″ By 1 prime and 1 ſecond.

1 — 04 — 08 — 06
01 — 04 — 08 — 06

1 — 06 — 01 — 02 — 06 *Facit.*

Rule 6.

To multiply any number of integers, primes, ſeconds, thirds, &c. by 12 integers.

Place the primes, ſeconds, thirds, &c. one place forward towards the left hand, and then multiply the integers by 12, and add it up.

1. *Ex.*

1. *Example.*

Multiply 14 — 10′ — 08″ By 12 integers.

10 — 08
168

178 — 08 · *Facit.*

2. *Example.*

Multiply 0 — 8′ — 10″ By 12 integers.

8 — 10 — *Facit.* viz. 8 integers 10 primes.

Rule 7.

To multiply any mixt number of integers, primes, ſeconds thirds, &c. by a mixt number of integers, primes, ſeconds, thirds, &c. the integers of the multiplicand, nor of the multiplyer, not exceeding 12.

This may be performed two ways.

Firſt, multiply the ſumme given by the integers of the multiplier, beginning at the leaſt Denomination, and carry one for every 12 to the next, and then multiply by the primes, ſeconds, &c. according to former direction, and add their products together.

1. *Ex-*

1. Example.

Multiply 8 — 04′ — 06″ By 6 — 03′.

50 — 03 — 00

2 — 01 — 01 — 06

52 — 04 — 01 — 06 *Facit.*

2. Example.

Multiply 12 — 08′ — 10″ By 8 — 06 — 06.

101 — 10· — 08·

6 — 04 — 05

06 — 04 — 05

108 — 09 — 05 — 05 *Facit.*

3. Example.

Multiply 5 — 03′ — 03″ By 3 — 05′ — 07″.

15 — 09· — 09·

1 — 09· — 01

0 — 05 — 03 — 3·

02 — 07· — 07 — 6

00 — 05 — 03 — 03

18 — 03 — 02 — 01 — 09 · *Facit.*

Or

Or Secondly, Place the integers under the last place, and then the primes, seconds, &c. following to the right hand, and then do as you have been taught in the second way of the third Rule: For here Note, That as multiplication of integers doth infinitely increase a number, so multiplication of Fractions doth infinitely decrease it.

1. *Example.*

Multiply	5	03′	03″	By 3 — 05 — 07		
		3	05	07		
		3	00	10	09	
	2	2	04	03		
15	9	09				
18	03	02	01	09 *Facit.*		

Note, *It matters not whether you multiply first by the Integers, or by the Duodecimalls, so you duly place your Figures.*

2. *Example.*

Multiply	6	08′	04″	3	By 4 — 08 — 04 — 02		
			4	08	04	02	
		01	01	04	08	06	
	2	2	9	05	00		
4	5	6	10	00			
26	9	05	00				
31	05	03	08	09	08	06 *Facit.*	

A Question.

How many square Inches are there in a peice of Board that squares 16 inches and $\frac{1}{2}$?

	Inches	*Parts.*
	16 ——	06
	16 ——	06
Or Thus.	96 ——	00
Inches.	168 ——	00
16 — $\frac{1}{2}$	8 ——	03
2		
33 half inches in one long inch	272 ——	03 *Facit.*

33
33

99
99

4) 1089 summe of the $\frac{1}{2}$ inches squared, which must be divided by 4, because every two $\frac{1}{2}$ inches squared made 4 half inches in squaring.

The same again $272\frac{1}{4}$

The same number of feet square are in a square perch of 16 $\frac{1}{2}$ foot.

Rule 8.

To multiply any number given in integers, primes, second, thirds, &c. by a whole number consisting of more than one figure, when any two or more numbers multiplyed into themselves, shall constitute or make up the multiplyer.

Having

Having found two or more numbers, which multiplied into themselves, ſhall make your multiplyer, multiply by one of thoſe numbers, and the product thereof by the other continually, and the laſt product ſhall be your deſire. As if you were to multiply by 48, ſay 6 times 8 is 48, then multiply firſt by 8, and the product thereof by 6, which numbers, *Viz.* 8, and 6, I call Ratio's. But if ſuch numbers cannot be found, then find ſuch two or more numbers, which being multiplyed into themſelves, ſhall come neareſt to, and leſs than the multiplyer; and having multiplyed by thoſe two or more numbers, multiply the given ſumme by the remaining number, and add that, and the laſt of the former products together, and that total ſhall give your deſire.

I would not have the Learner to ſlight this, becauſe he will find it to be of moſt excellent uſe.

1. *Example.*

Multiply—6 — 06′ — 08″ By 36.
6

39 — 04 — 00
6

236 — 00 — 00 *Facit.*

Here I ſay 6 times 6 is 36, therefore I multiply by 6, and the product thereof by 6 again.

2. *Example.*

Multiply–8 — $04'$ — $04''$ By 38.
6

50 — 02 — 00
6 — 2

301 — 00 — 00
16 — 08 — 08

317 — 08 — 08–*Facit.*

Here I ſay 6 times 6 is 36, and 2 is 38 therefore I multiplyed by 6 and the product by 6, and the given ſumme by 2, and added the two laſt products together.

3. *Example.*

Multiply– 9 — $07'$ — $05''$ By 67.
9

86 — 06 — 09
7 — 4

605 — 11 — 03
38 — 05 — 08

644 — 04 — 11 *Facit.*

Rule.

Rule 9. Generally.

To multiply any number given in integers, primes, ſeconds, thirds, &c. by any number of integers, primes, ſeconds, &c. whatſoever.

This may be performed two ways.

Firſt multiply one whole number by the other, then multiply the whole multiplicand by the primes of the multiplier, then for the ſeconds in the multiplier, take $\frac{1}{12}$ of the whole multiplicand, and croſs out all the figures of the $\frac{1}{12}$ ſo taken, (becauſe you muſt omit them in adding) and then take ſuch a part or parts of that $\frac{1}{12}$, as the ſeconds in the multiplyer do make; &c. and then you have nothing more to multiply, but the whole number in the multiplyer, by the primes, and ſeconds, &c. in the multiplicand, which to perform, firſt multiply by the primes, & then for the ſeconds take the $\frac{1}{12}$ of the whole number, (croſſing it) and then take ſuch a part or parts thereof, as the ſeconds do make, &c.

1. *Example.*

Multiply – 6789 — 06′ — 04″ By 24 — 03′ — 08″.
24 — 03 — 08

27156
13578·
1697 — 04 — 07
~~565~~ — ~~09~~ — ~~06~~ — ~~04~~
282 — 10 — 09 — 02
94 — 03 — 07 — 00 — 08
12 — 00 — 00 — 00 — 00
~~2~~ — ~~00~~ — ~~00~~ — ~~00~~ — ~~00~~
0 — 08 — 00 — 00 — 00

165023 — 02 — 11 — 02 — 08 *Facit.*

---

2. *Example.*

Multiply 6378 — 04′ — 08″ By 349 — 06′ — 09″.
349 — 06 — 09

57402
255120
1913406
3189 — 02 — 04
~~331~~ — ~~08~~ — ~~04~~ — ~~08~~
265 — 09 — 02 — 04
132 — 10 — 07 — 02
116 — 04 — 00 — 00
~~29~~ — ~~01~~ — ~~00~~ — ~~00~~
14 — 06 — 06 — 00
4 — 10 — 02 — 06

2229645 — 06 — 09 — 06 *Facit.*

Or Secondly, and more briefly, multiply one whole number by the other, and then the whole multiplicand by the primes of the multiplier, as before. Then to multiply by the seconds in the multiplyer, put the primes into seconds, and find what part or parts of those seconds, the seconds you are to multiply by, do make, and take the part, or parts so found accordingly of the product of the primes: And when you come to multiply the whole number in your multiplier, by the primes, and seconds of your multiplicand, do the same, &c.

3. *Ex-*

1. *Example.*

```
                      '      ''
* Multiply 6789 — 06 — 04 By 24 — 03 — 08
                24 — 03 — 08
   ______________________________________
      27156
     13578 ·
       1697 — 04 — 07
        282 — 10 — 09 — 02
         94 — 03 — 07 — 00 — 08
†        12 — 08 — 00 — 00 — 00
   ______________________________________
X     165023 — 02 — 11 — 02 — 08 Facit.
   ______________________________________
```

* *Note*, I might have multiplyed by the Ratio's of 24, *viz.* 6, and 4.

† *Note again*, when I came to multiply 24 by 6 primes, and 4 seconds; for the 6 primes I took the $\frac{1}{2}$, *facit* 12, then for the 4 seconds, I put the 6 primes into seconds, *facit* 72, then I found 4 might be had in 72, 18 times, then 18 being more then 12, I reckoned it $\frac{12}{18}$ or $\frac{2}{3}$ which makes 8 primes.

X *Note again*, That taking a part or parts of a part being of most excellent and general use, I would have the Learner to be very perfect in it. And for his encouragement (that he may not think it difficult) I would have him know, that he will never have occasion for more than 1,2,3,4,5, or 6. parts to take the part or parts of. And though I have made it plain already above in the first Note, yet for his better understanding thereof, I will give him one Example that he may know how to find what part of a part any part may be. As suppose having multiplyed by 4 primes, you were

to multiply by 8 ſeconds, now the quere is what part 8 ſeconds is of 4 primes, which to find, ſay 4 primes is 48 ſeconds, then ſay how many times 8 in 48, *facit* 6 times, and ſo you have found that 8 ſeconds is the 6th. part of 4 primes, and then if you take the 6th. of the product of 4 primes, you have your deſired product of 8 ſeconds, and the ſame Rule holds for any other parts *ad infinitum*.

2. *Example.*

```
                         '       ''
Multiply - 6378 —— 04 —— 08 By 349 —— 06 —— 09
            349 —— 06 —— 09
——————————————————————————————————————————————
          57402
         255120
        1913400
           3189 —— 02 —— 04
            398 —— 07 —— 09 —— 06
            116 —— 04 —— .. —— ..
             19 —— 04 —— 08 —— ..
——————————————————————————————————————————————
        2229645 —— 06 —— 09 —— 06 Facit.
——————————————————————————————————————————————
```

DIVISION.

Rule 1.

To Divide by any one of the 9 Digits.

FIrſt, if it be by any number, by which you work as by an Aliquot part of 12, as by 2, 3, 4, 6, 8, then your

your work will be the ſame with that in multiplying by thoſe aliquot parts ; ſo will it alſo be if you are to divide by 12.

Example.

Divide—	8	08′	09″	By 4
Facit.	2	02	02	03.

Note, I take the 4th. and that which remains at every place are 4ths.

Secondly, If it be by any number by which you cannot work as by an aliquot part of 12, as by 5, 7, 9, then you muſt take the 5th. the 7th. or the 9th. as before, but what remains (being no aliquot parts) you muſt put into the next denomination, adding thereto the number ſtanding in the place of that denomination into which you have brought it, and then take the 5th. the 7th. or the 9th. of that continually, as occaſion requireth. *Note*, you may divide alſo by 11 the ſame way.

1. *Example.*

Divide-	84	08′	10″	By 5.
Facit-	16	11	04	04 $\frac{4}{5}$

2. *Example.*

Divide-	84	08′	10″	By 7.
Facit	12	01	03	01 $\frac{5}{7}$

4. *Ex-*

3. Example.

Divide-84 — $08'$ — $10''$ By 9.

Facit. 9 — 04 — 11 — 9 — 04.

4. Example.

Divide-84 — $08'$ — $10''$ — **By** 11.

Facit. 07 — 08 — 05 $\frac{3}{11}$.

Rule 2.

To divide any number of integers, primes, &c. by any number of integers, primes, &c. and the quotient to contain but one figure of integers.

You have nothing more here to do, but to find how many times the divisor is contained in the dividend.

Example.

Divide-16 — $08'$ — $04''$ By 7 — $6'$ — $9''$

7 — $6'$ — $9''$) 16 — 08 — 04 (2 — $02'$ — $05''$ — $10'''$ *facit*

* 1 — 06 — 10 — 00

— 03 — 08 — 06 — 00

— 6 — 08 — 03 — 00

— 04 — 07 — 06 *Remain*

Note,

* Note, *When your dividend is one place more than your divisor, you must always put the two first places together, and into Duodecimals, when you are to make your demand for the first figure of your Divisor.*

First I found how many times 7 I could have in 16 *facit* twice, therefore I placed 2 in the quotient, then I multiplyed the divisor by the 2, saying 2 times 9 is 18, which being 6 above the duodecimal, I said 6 from 4 I cannot, but 6 from 12 and there remains 6, which added to the 4 makes 10, which 10 I sate down, then I said 2 times 6 is 12, and 1 that I carried, and 1 that I borrowed is 14, which being 2 above the duodecimal I said 2 from 8, and there remain'd 6, which 6 I also sate down, then I said 2 times 7 is 14, and 1 that I carried is 15, from 16, and there remain'd 1, with which the work was ended for the integers in the quotient. Then to find the primes, I added, a cypher to the remain, to increase it one place, and said, how many times 7 in 18, (which is 1 integer, and 6 primes) *facit* 2, then I multiplied and subtracted as before, and in the same manner it may be brought as low as you please, as you may see in the Example.

But it may be easier to some, to set down the multiplication, and then to subtract it. And therefore I shall so divide the same sum for an Example.

7-6-9) 16 — 08 — 04 (2-2′-5″-10‴ *Facit.*
15 — 01 — 06
1 — 06 — 10 — 00
1 — 03 — 01 — 06
· — 03 — 08 — 06 — 00
03 — 01 — 09 — 09
· — 06 — 08 — 03 — 00
06 — 03 — 07 — 06
· — 04 — 07 — 06 *Rem.*

The

The foregoing Example proved by Multiplication.

2	2′	5″	10‴		
			7	6	9
	1	7	10	4	6
1	1	2	11	0	6
15	5	4	10	7	
			4		
16	8	4	·	·	·

Rule 3.

To divide a leſſer number of integers, primes, &c. by a greater number of integers, primes, &c.

Add one place more to the dividend by making a cypher, and then count the integers of the diviſor ſo many primes, and the primes of the diviſor ſo many ſeconds, &c. then put thoſe primes into integers, ſetting down the odd primes (if any,) Then ſee how many times thoſe integers, primes, &c. are contained in the dividend, as you have been taught, and count the quotient primes, &c. And ſo you may do until you have brought it to the leaſt denomination deſired.

1. *Ex-*

1. *Example.*

Divide-24 — 09′ — 10″. By 48 — 10′ — 10″

4 — 00 — 10 — 10 Divisor.

(4-00′-10″-10‴)

(0-06-01-1- 0- 0- 9 Quotient.

24-09-10-00

·-04-05-00-00

·-04-01-02-00

·-00-03-02-00-00-00

·-01-03-10-06 *Remain.*

Note, When I came to ask how many times 4 I could have in 0 — 0 — 0 — 0 — 3 — 2, I had two ways to find it, *viz.* either by saying how many times 4 I could have in 38, *facit* 9, or by saying $\frac{3}{4}$ is 9, &c. how ever it must be tryed whether 9 times will pass the dividend or not, *&c.*

2. *Example.*

Divide-38-09′-11″-08‴ By 54-07′- 9″- 9‴.
Divisor

(4-06-07-09-09   4-06-07 -9 -9 div.

dividend 38-09-11-08-00 (0-08′-06″-03‴-09⁗-05 quot.

2-04-09-02-00-00

·-01-05-03-01-06-00

·-03-07-02-00-09-00

·-02-02-02-05-03-00

·-03-05-02-02-03. *Remain.*

Rule 4.

Rule 4.

To divide by any number of integers, consisting of two or more numbers, when other two or more * numbers may be found, which multiplyed into themselves, shall constitute the same divisor.

* Note, *The way to find such numbers is to reduce the Divisor to a unite, and the numbers whereby it was reduced, are the numbers to work by, for they multiplied into themselves will make up the Divisor. And those numbers I call* Ratio's.

The manner of reducing.

$$\textit{Ratio's}\left\{\begin{array}{c}12\\ \\6\\ \\4\end{array}\right\}\begin{array}{c}288\\ \hline 24\\ \hline 4\\ \hline 1\end{array}$$

Divide by the first of those numbers so found, and the quotient of that by the next, &c. and the last quotient shall be your desire.

1. *Example.*

Divide-84 —- 08 — 10 By 36

14 — 01 — 05 — 08

Facit ·2 — 04 — 02 —- 11 — 4

Here I say 6 times 6 is 36, so 6 and 6 are the Ratio's.

2. *Ex-*

2. Example.

Divide - 1000 —— 08 —— 06 By 288

·83 —— 04 —— 08 —— 06

13 —— 10 —— 09 —— 05

Facit 3 —— 05 —— 08 —— 04 —— 03

Here the ratio's are 12—6—4 as is mentioned before.

Rule 5.

To divide any number by an irrational number, *viz.* by ſuch a number as for which no ratio's may be found.

Divide by the ratio's of the neareſt rational number leſſe; then multiply the exceſs by the integers of the quote, and ſubtract the product from the quote and the remain.

1. *Example.*

Divide - 78 —— By 19.

Now 3 times 6 is 18. So 18 is the neareſt rational number leſs.

13 ———— 00

4 ———— 06

4

4 ———— $\frac{2}{19}$ quote

Note, what remains on the firſt ratio, multiply by the laſt ratio, and what remains on the laſt ratio, multiply by the firſt, adding thereto as many as you can take of the laſt ratio, out of the remain of the firſt.

Note,

Note again, I took the 6th. *facit* 13. then I took the 3d. of that 13 *facit* 4-6. for the quote by 18 : then 1 being the excess, I said 4 times 1 is 4, which subtracted from 6, the remain was 2. so the true quote makes $4\frac{2}{13}$.

Again.

```
Divide-89 —— By 23
  ————— —————
    12 ———— 15
  ————— —————
     4 ———— 05
              8
  ————— —————
     3 ———— 20 Quote.
```

Now 3 times 7 is 21, so the excess is 2.

If there happen 3 or 4 figures of integers in the quote, see how many times the quote makes the divisor as in the Example following: 3 times 37 makes 111, which with the remain 33 makes 144 from which 126 (the quote) subtracted, leaves 18 for the remain, and 3 (37 being contained 3 times in 126) subtracted from the quote, leaves 123. So the true quote is $123, \frac{18}{37}$.

```
37)   4569
 3
————  ——————————
111   761 —— 18
 33
————  ——————————
144   126 —— 33
126     3
————  ——————————
 18   123 —— 18/37 Quote.
```

The ratio's are 6 & 6.

Rule 6.

Rule 6.

To divide any number of integers, by any number of integers, primes, ſeconds, &c.

Set your dividend, and draw a crooked line at the begining for the diviſor, and another at the end for the quotient, as in common Diviſion, then draw a line under the dividend, and ſet under the line the firſt part of the dividend, and make as many places (by cyphers) as you have occaſion for, for primes, ſeconds, &c. Then find how often the diviſor may be found in that part of the dividend ſo ſate down, and the number found place in the quotient, then multiply the diviſor by that number, and ſubtract as in common diviſion, and if any primes, ſeconds, &c. remain, multiply them by 10, and carry the integers which they make (if any) in mind, then point the next figure of the dividend, and add the integers in your mind thereunto, and ſet it down, and the remaining integers of your former work to the left hand thereof; but if the integers you carryed in mind, and the figure pointed and taken down, amount to or exceed 10, you muſt add a unite to the laſt figure that remained of your former work, and ſo you have a new dividend, then find how oft the diviſor is contained therein, and work as before. And ſo you muſt do until you have gone through all the dividend.

Note, *This Rule is alwayes to be obſerved if the quotient muſt conſiſt of two or more figures of integers, but if the quotient muſt conſiſt but of one figure of integers, then you muſt work according to the ſecond Rule.*

1. *Example.*

Divide--75896 By 24—09′

24—09) 75896 (3066 $\frac{12—06}{24—09}$ quotient.

75—00

· :—09   24—09

15—06   New divisor. 2—00—09; to find the value of the Fraction; which is done according to the Third Rule.

164—00

15—06

161—00

Remain—12—06

primes

20-00-09)12—06—00 (0—6—00—08—08

· ·—01—06—00—00

· ·—01—06—00—00

· ·—01—06—00

integers primes seconds thirds fourths.
The Answer is 3066—06—0—08—08 and somewhat more.

2. *Example.*

Divide —— 98764 By 24 —— 08′

```
                       ′    ″    ‴   ⁗
24——08) 98764 (3003-11-04-02-07
        ———— ——  1
          98—00
        ————————
          24—00
        ————————
         247—00
        ————————
         · · ·—04
        ————————
            9—04
        ————————
           97—04
        ————————
New divis. 2-00-08)23—04—00
                   ——————————
                    ·—08—08—00
                   ——————————————
                        ·—05—04—00
                      ——————————————
                           01—02—08—00
                          ——————————————
                            ·—· · —03—04
```

Note, *Sometimes it happens in dividing this way that you may take* 10 *times the Divisor, and then you must place the unite under the former figure in the quotient, as you may see* 1 *placed under the* 3 *above, therefore the* 3 *must be accounted* 4, *and the quotient read* 4003, *&c. And sometimes you may take* 11 *times the divisor, (but never more) and then you must place* 1 *under the former figure, and the other* 1 *in its proper place as you may find it to happen in the next Example.*

3. *Example.*

```
                      '  ''  '''
1 — 8 — 1 — 6) 69 — 5 — 4 (31-4-10-8
               ———————————— 1
                6—0—0—0
               ————————————
                ·—11—7—6
               ————————————
                19—01—7—0
               ————————————
                ·—08—2—6—0
                ——————————————
                  01—6—0—0—0
                 ——————————————
                  ·—1—2—9—0—0
                 ——————————————
                     ·—1—4—0—0
```

Note, *You must take in the Duodecimalls in their proper places, as primes in the place of primes*, &c.

So that the quotient must be read 41-4-10-8 (with ' '' ''' over 4, 10, 8).

This last Example proved by Multiplication.

```
41 — 04 — 10 — 08
 1 — 08 — 01 — 06
————————————————————
41 — 04 — 10· — 08·
20 — 08· — 05 — 04
 6 — 10· — 09· — 09· — 04.
          3 — 05 — 04 — 10· — 08·
          1 — 08· — 08 — 05 — 04
                      1 — 04
—————————————————————————————————————
69 — 05 — 04 — · · — · · — · · Proof.
—————————————————————————————————————
```

Rule 7.

Rule 7. *Generally.*

To divide any number given in integers, primes, ſeconds, &c. by any number of integers, primes, ſeconds, &c.

Work as you were taught in the foregoing Rule, and when you have gone through all the integerial figures of the dividend, except the laſt, multiply the remaining Duodecimalls by 10 (as you did before,) and add in the Duodecimalls of the dividend, &c.

1. *Example.*

Divide—56 — 05′— 08″ By 2—03′—04″

2 — 3 — 4) 56—05—08 (24—09′—06″ *Facit.*

2—03—04

5—00—00

49—07—00

·—05—04 6—02—04—6·

8—03—2

10—11—00 4

New diviſor

0—2—3—4.)1—09—08—0 *Remain.*

·—1—2—0—0|56—05—08—. proof.

··—··—04—00

2. *Example.*

Divide —— 569— 7— 9 By 3— ′4— ″8.

3—4—8) 569— 7— 9 (168-01-01-03 *Facit.*

5-00-00	3-04-08
1-07-04	504-03-03-09
22-01-04	56-00-04-05
1-09-04	7-00-00-06-07-06
27-05-01	2-04-00-02-02-06
	1-02-00
	569-07-09-.. -..-.. *proof.*

New divis.
0-3-4-8). .-03-09-00 *Remain.*

. .-04-04-00

. .-11-04-00

01-02-00

Note, *When I came for the proof to multiply by* 8 *seconds, I put the* 4 *primes into seconds, saying* 4 *times* 12 *is* 48, *then I askt how many times* 6 *in* 48, facit 8 *times, therefore I took the* 8*th. for* 6 *seconds of the product of* 4 *primes, and for the other two seconds I took the* 3*d. of the product of* 6 *seconds. But it might have been done in one line by taking the* 6*th. of the product of* 4 *primes,* 8 *seconds being the* 6*th. thereof.*

3. *Example.*

Divide - 165023—02′—11″—02‴—08⁗ By 6789-06′-04″

12)6789-6-4)165023—02—11—02-08(24-03-08 *Facit.*

565-9-6-4)16502—00—00

2922—11—04

29232—08—03—02—08

New divisor

12)565-9-6-4) · 2074—06—11—02—08

47-1-9-6-4)·377—02—04—02—08

. . .—. .—. .—. .—. .

Note, *As long as you have more than one figure of integerial parts in the Fraction, and more than one figure of integers in the Divisor, you must continually take the* 12th. *of the Divisor for a new one to find the value of the Fraction.*

4. *Example.*

Divide - 2229645—6—9—6 By 6378—4—8

6378—4—8)2229645—6—9—6(349—06—09 *Facit.*

531-6-4-8)22296—00—00

3160—10—00

31612—04—00

6098—09—04

60993—04—01

531—6—4—8.)3587—10—01—06

44-3-6-4-8) 398—07—09—06—00

. . .—. .—. .—. .—. .

Though this way of dividing may seem tedious at first, yet a little practice will make it as easie and familiar as any other way whatsoever.

These two last Examples are the products of the two last Examples in multiplication, to shew that as multiplication proves Division, so Division proves Multiplication.

Here followeth four necessary Questions.

1. *Question.*

WHat number is that which shall divide $818\text{-}\overset{\prime}{06}\text{-}\overset{\prime\prime}{00}\text{-}\overset{\prime\prime\prime}{07}$ that the Quotient may be $25\text{—}\overset{\prime}{04}\text{—}\overset{\prime\prime}{02}$?

Resolution.

Divide 818—6—0—7 By 25-4-2, and the quotient will be the Divisor sought.

Operation.

```
25—4—2)818—06—00—07(32—03—06
       81—00—00
        4—11—06
       58—01—00
2—1—4—2) 7—04—08—07
        1—00—08—01—00
```

So

So the number by which if you divide 818—6—0—7 that the quotient may be 25—4—2. will be 32-3-6.

Instance.

32—3—6)818 — 6 — 0 — 7 (25—04—02.

81 — 00 — 00

16 — 05 — 00

172 — 08 — 00

2—8—3—6) 11 — 02 — 06 — 07

· · — 05 — 04 — 07 — 00

· · — · · — · · — · ·

Note, *Any operation in Division may be proved by Division, as here doth appear, for if after your Division is ended you divide the dividend by the quotient, the new quotient thence arising, will be equal to the divisor of the first operation. But if any thing remain on the first division, subtract it from the dividend, and divide the remain by the quotient.*

2. *Qustion.*

What number is that which if divided by 25 — 4′ — 2″ the quotient shall be 32 — 3′ — 6″?

Resolution.

Multiply 25 — 4′ — 2″, By 32—3′—6″, and the product will be the dividend enquired for.

Opera-

Operation.

	′	″	‴
25	4	2	
32	3	6	
202	9	4	
811	1	4	
6	4	0	6
1	0	8	1
Facit 818	6	0	7 for the dividend (as before.

3. *Question.*

What number is that which ſhall multiply $35 — 4' — 2''$, that the product may be $818 — 6' — 0'' — 7'''$?

Reſolution.

Divide 818 —— 6 —— 0 —— 7, by 25 —— 4 —— 2, and the quotient gives the deſired Multiplyer, *Facit* $32 — 3' — 6''$ as above.

4. *Queſtion.*

What number is that which if multiplyed by $25 — 4' — 2''$, the product may be $818 — 6' — 0'' — 7'''$?

Reſolution.

Divide 818 —— 6 —— 0 —— 7, by 25 —— 4 —— 2, and the quotient is the multiplicand ſought, *Facit* $32 — 3' — 6''$. as before.

8. *Rule*

8. *Rule*

To perform oftentimes *Multiplication*, and *Division*, by one operation, as if I were to multiply by one number, and divide by another.

First, Observe how much the Divisor is more or less than the multiplier, and if both can be divided by that difference, and nothing remain, it may thus be performed, *viz.* Seek how often that number that will so divide is contained in the divisor, and by the number so found divide the multiplicand. And when the divisor is more than the multiplier subduct the quotient from the multiplicand, and when it is less, add it thereunto, and you have your desire.

Secondly, Observe whether the divisor may be divided by the multiplier, or whether the multiplier may be divided by the divisor, and nothing remain; then may you divide by the quote of the former, or multiply by the quote of the latter. And both these numbers (the former, and this latter) so found for operation, may be called compound Ratio's, as resulting both from the multiplier, and divisor.

Examples of the first kind.

1. Multiply 84—08$'$—10$''$ By 12 & divide the product by 16.
21—02—02—06

63—06—07—06 *Facit.*

Here the *Divisor* is more than the *Multiplier.*

2. Multiply 84—08—10 by 16, & divide the product by 12.
28—02—11—04

112—11—09—04

Here the *Divisor* is less than the *Multiplier.*

Exam-

Examples of the second kind.

1. Multiply 98—10—08 by 12, & divide the product by 36.

32—11—06—08 *Facit.*

Here the *Divisor* may be divided by the *Multiplier.*

2. Multiply 56—09—09 by 36, & divide the product by 12.

170—05—03 *Facit.*

Here the *Multiplier* may be divided by the *Divisor.*

Note yet Two things more, viz.

First, If the divisor be so great, and the multiplier so small that it cannot be performed by this Rule, as if you were to multiply by 5, and to divide the product by 100, then see what part of 100 may be divided by 5, and you will find it to be the $\frac{1}{5}$, which is 20, and 5 (the multiplier) is the $\frac{1}{4}$ of 20, therefore take the $\frac{1}{5}$, and then the $\frac{1}{4}$ of that gives your desire.

Example.

Multiply 84—08′—10″ by 5 and divide the product by 100.

16—11—04—04 $\frac{4}{5}$

4—02—10—01. $\frac{1}{5}$ *Facit.*

Secondly,

Secondly, On the contrary, if the multiplier in like manner be more than the divisor, then take such a part of the multiplier as may be divided by the divisor, and multiply the given sum by that part, and then multiply that product by the part of that part (so taken) which the multiplier is of. As if you were to multiply 4 — 02 — 10 — 01 – $\frac{1}{5}$ by 100, and divide the product by 5 : you know 20 is the $\frac{1}{5}$ of 100, and 5 is the $\frac{1}{4}$ of that, therefore multiply by 5 and the product by 4.

Operation.

$$
\begin{array}{l}
4 — 02' — 10'' — 01''' \tfrac{1}{5} \\
\hline
21 — 02 — 02 — 06\text{-}0 \\
\hline
84 — 08 — 10 — 00\text{-}0 \\
\hline
\end{array}
$$

This produceth the first given sum again being the contrary.

Though the fourth Rule in Division, and this 8th. be not general, yet you will find them very profitable, for they come very often into use and practice, and therefore I advise you not to slight them.

Rule 9.

To divide any number given in integers and Duodecimals by a vulgar Fraction.

Here Note, that there are 3 cases to be considered, *viz.*

First, If the Numerator be but 1, as $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$, &c. then multiply by the denominator, as you were taught before for a whole number.

Second-

Secondly, But if the numerator be 2 - 3 - 4, &c. then multiply by the denominator as before, and divide by the numerator, as you were taught before for a whole number.

Thirdly, But if the Fraction be such as you may find a compound Ratio, between the numerator and the denominator, then you may work as you have been taught in the first part of the 8th. Rule.

Examples of the First Case.

$$1.\ \text{Divide}-32 — 10' — 06'' \text{ By } \tfrac{1}{2}$$

$$\begin{array}{r} 32 — 10' — 06'' \\ 2 \\ \hline 65 — 09 — 00 \textit{ Facit.} \end{array}$$

$$2.\ \text{Divide}-54 — 11' — 08'' \text{ By } \tfrac{1}{5}$$

$$\begin{array}{r} 54 — 11' — 08'' \\ 5 \\ \hline 274 — 10 — 04 \textit{ Facit} \end{array}$$

Examples of the Second Case.

$$1.\ \text{Divide}-78 — 08' — 06'' \text{ By } \tfrac{5}{8}$$

$$\begin{array}{r} 78 — 08' — 06'' \\ 8 \\ \hline 5)\ 629 — 08 — 00 \\ \hline 125 — 11 — 02\ \tfrac{2}{5} \textit{ Facit.} \end{array}$$

2. Di-

2. Divide-56—09′—10″ By $\frac{3}{5}$

5

3) 284—01—02

94—08—04—08 *Facit.*

Examples of the Third Case.

1. Divide-32—10′—06″ By $\frac{2}{3}$

16—05—03

49— 3— 9

Here the compound Ratio betwixt 2 and 3 is 2, therefore I divide by 2, which is performed by taking the $\frac{1}{2}$, &c.

2. Divide-32—10′—06″ By $\frac{3}{4}$

10—11—06

43—10—00 *Facit.*

Here the compound Ratio betwixt 3 and 4 is 3, therefore I take the 3d. &c.

3. Divide-32—10′—06″ By $\frac{4}{5}$

8—02—07—06

41—01—01—06

Here the compound Ratio is 4.

10. *Rule.*

10. *Rule.*

To Divide Integers by Duodecimals.

Reduce the Integers into Duodecimals, and then divide as you have been taught.

1. *Example.*

Divide-8 By 0—06′—06″

8

12 (14′-9 01—06 / 06—06 *Facit*. That is to say, 0-6-6 are contained in 8 Integers 14 times, 9/12, and more the Fraction.

0—6—6) 96 primes

9—00

2—06

31—00

5—00—00

—01—06

2. *Example.*

Divide 10—08′—04″ By 0—08′—09″

12

0-8-9.) 128—04 (14—08′ *Facit.*

12—00 | 7—04

2— 05—04

07—04

03—08

3—03

40—10 | 10—08—04 *Prov. by (mult.*

5—10—00

REDUCTION.

REDUCTION.

THere are but 4 particulars that are here neceſſary to be ſpoken too, *viz.*

Firſt, How to reduce Vulgar Fractions into Duodecimals.

Secondly, How to reduce decimals into duodecimals.

Thirdly, How to reduce a duodecimal Fraction into its loweſt parts.

Fourthly, How to reduce duodecimals into decimals.

For the firſt, To reduce Vulgar Fractions into duodecimals.

Multiply the numerator of the Vulgar Fraction by 12 and divide the product by the denominator.

Example.

Reduce $\frac{3}{5}$ *into Duodecimalls.*

$$\begin{array}{r} 12 \\ 3 \\ \hline 36 \end{array} \qquad 5\,)\,\begin{array}{c} 36 \\ \hline 1 \end{array}\,(\,7'$$

Facit, 7 primes, and $\frac{1}{5}$ which $\frac{1}{5}$ may be brought into the next Duodecimall, by ſaying 12 times 1 is 12, and 12 divided by 5 makes 2 ſeconds $\frac{2}{5}$, &c.

For the Second, To reduce decimals into duodecimalls.

Multiply the decimal Fraction by 12, and cut off as many figures at the right hand, as there are places in the decimal fraction, ſo to be reduced, and ſo you muſt do continually until you have brought it to the loweſt termes you deſire.

Example.

I would reduce 862, decimalls into doodecimalls.

	862
	12
10	344
	12
4	128
	12
1	536

Facit 0-10′-04″-01‴ duodecimalls, (&c.

For the Third, To reduce a duodecimall fraction into its loweſt parts, this being performed by the Third Rule in Diviſion, I refer you thereunto.

For the Fourth, To reduce duodecimalls into decimalls.

Multiply the duodecimalls by 10 duodecimally, and cut off the firſt duodecimall at the left hand, for the firſt decimall: Then multiply the figures cut off at the right hand by 10 duodecimally again, and cut off the firſt duodecimall at the left hand for the next decimall, and ſo do continually until you have brought it as low as you deſire.

Example.

Reduce 0-8-4-3 duodecimalls into decimalls.

	0-8-4-3
	10
6	11-6-6
	10
9	7-5-0
	10
6	2-2-0
	10
1	9-8-0

Facit. 6961 decimalls.

Of

Of Mean Proportions.

THere are 3 mean Proportions between numbers mentioned by Mathematical Authours, *viz. Arithmetical, Geometrical, and Musical.*

Arithmetical Mean Proportion, is the half summe of the two extreams added together, as thus, the Arithmetical mean between 25 and 14 is 19 $\frac{1}{2}$, being the half of 39, the summe of 25, and 14 added together.

The Geometrical mean Proportion is the square root of the product of the two extream numbers multiplyed one by the other.

The Musical mean Proportion is less than the *Geometrical*, and is thus found, *viz.* As the Arithmetical mean to one extream, so is the other extream to the Musical mean.

But the *Geometrical mean Proportion*, is of most use for the present design, and therefore being not to be found without the square root, I shall in the next place shew enough thereof very briefly for the present purpose.

Square Root.

FIrst, The Extraction of the Square Root of any number is to find out another number that being squared or multiplyed by it self, the product is equal to the first number propounded.

Thus, If 144 be a number propounded to find the Square Root of, you ſhall find that 12 is the ſquare root thereof, for 12 ſquared, *viz.* multiplyed by 12 produceth 144, the number firſt propounded.

Secondly, Square numbers are *Single* or *Compound*; *Single*, whoſe roots conſiſt but of 1 figure, or *Compound*, whoſe roots conſiſt of more than one figure.

Root	ſquare
1	1
2	4
3	9
4	16
5	25
6	36
7	49
8	64
9	81
10	100

The ſingle ſquare numbers are only theſe 9 under 100, and the ſingle digits from 1 to 9 are the ſquare roots of thoſe ſquare numbers, as in the Margent, and muſt (as the multiplication Table) be learned by heart.

All others above 100 are Compound ſquare numbers.

Thirdly, All numbers are either juſt ſquare numbers, whoſe roots come to the ſame without any remainder, as 81 whoſe ſquare root is 9, and 9 ſquared is 81. Or elſe called Surd numbers, whoſe roots being ſquared, will not come to the firſt number, without ſome fraction or remainder; thus 52 is a Surd number, whoſe neareſt root is $7\frac{3}{15}$. How to find the valuation of the remainder, I ſhall ſhew hereafter.

Firſt, I ſhall begin with ſingle numbers, both Square and Surd.

Firſt, Of ſingle Square Numbers.

Here you have nothing to do but to take the root as you find it in the Table. You may draw a quotient for the root as you do in diviſion, or rather, you may draw a line under the ſquare number, and ſet the root under it.

Example.

What is the ſquare root of 49? Conſider what number is that which multiplyed into it ſelf will make 49, for that number

number is the root thereof, which if you cannot tell, look in the Table, and you will find it to be 7, 7 times 7 being 49.

49

Quotient. 7 the ſquare root,

Secondly, Of ſingle Surd numbers.

Firſt, find the neareſt root as before, and ſubtract the ſquare thereof from the Surd number given, and ſet the remain under the root, with a line drawn betwixt them, then to find the value of the remain, add two places (by making cyphers) to the remain, then ſet the double of the root under the firſt of the two places you added towards the left hand, then put the integers into primes, and find how often you may have the double of the root (ſo ſate,) in thoſe primes, and the number found place both in the quotient in the place of primes, and under the laſt of the two places you added at the right hand, then multiply and ſubtract as you did in diviſion.

But here note, That you muſt be ſure to take no more; than when the double of the root, and the figure ſate at the right hand ſhall be ſo mutipliped, the product may be ſubtracted, for if the product ſhall exceed the number from whence you are to ſubtract; you muſt then take leſs. But ſtill you muſt take as much as you can. And when you have ſo done for the primes, if a fraction remains again, you muſt take the ſame order continually, untill you have brought it to the leaſt denomination you deſire, putting the duodecimals into the next denomination, &c.

1. *Example.*

What is the ſquare root of 17?

17

4 — 01 — 05 — 08 . *ſquare root.*

1 — 00 — 00
8 — 01

. — 3 — 11 — 00 — 00
8 — 02 — 05

. — 5 — 11 — 11 — 00 — 00
8 — 02 — 10 — 08

. — 5 — 11 — 10 — 00 — 08. *rem.*

2. *Example.*

I demand the ſquare root of 54.

54

7 — 04 — 02 — 02 : *ſquare root.*

5 — 00 — 00
1 — 02 — 04

. — 02 — 08 — 00 — 00
1 — 02 — 08 — 02

. — 02 — 07 — 08 — 00 — 00
1 — 02 — 08 — 04 — 02

. — 02 — 03 — 03 — 08 *remain*
7 — 04

7 — 04 — 02 — 02
7 — 04 — 02 — 02

51 — 05 — 03 — 02
2 — 05 — 04 — 08 — 08
1 — 02 — 08 — 04 — 04
1 — 02 — 08 — 04 — 04
2 — 03 — 03 — 08

54 — · · — · · — · · — · · — · · — · · *Proof.*

3. *Example.*

I demand the square root of 72—08—08—01—04.

72 — 08 — 08 — 01 — 04

8 — 06 — 04 : *square root.*

8 — 08 — 08
1 — 04 — 06

Note, *I put the duodecimalls in the places I were to add.*

· — 05 — 08 — 01 — 04
01 — 05 — 00 — 04

· · — · · — · · — · ·

8 — 06 — 04
8 — 06 — 04

Whereby it appears to be a square number.

68 — 02 — 08
4 — 03 — 02
02 — 10 — 01 — 04

72 — 08 — 08 — 01 — 04 *Proof.*

2. *Of Compound numbers Square and Surd.*

First of Square Numbers.

When you would find the ſquare root of a compound number, whether ſquare or ſurd, you muſt prepare it with points, thus, *viz.* ſet a point over every other figure beginning with the firſt figure at the right hand, then draw two lines under the number for the quotient, and find the ſquare root of the firſt period (two figures being in every period after the firſt, which may have but one,) and place it in the quotient, then from that period ſubtract the ſquare of the root, and ſet down the remain, with the next period after it, and ſo you have as it were a new dividend; then to find the diviſor (in order to find the next figure in the root to be placed in the quotient,) double the root before found, and place it under the firſt figure to the left hand of the ſecond period which you ſate down, then find how often you may have the diviſor in the dividend, and the number found place both in the quotient, and under the laſt figure of the period or dividend, at the right hand; then multiply and ſubtract as in diviſion. But here you muſt Note (as befored directed,) that you take neither too much nor too little. And ſo you muſt do until you have ran through all the periods.

1. *Example.*

What is the ſquare root of 5476?

$\dot{5}4\dot{7}6$

Quotient 74 Square root.

576
144

...

2. *Ex-*

2. *Example.*

What is the ſquare root of 555025?

```
 . . .
555025
   . .
 ______
Quotient 745 Square root.
 ______
 650
 144
 ______
 ·7425
  1485
 ______
 . . . .
```

You may place a point under the laſt figure you take down, to prevent miſtake in ſetting.

2. *Of Surd Numbers.*

HAving gone through all the periods as before directed, to find the value of the remain you are to add two places to the remain continually, and then work as you have been taught in ſingle Surd numbers, onely with this difference; that you take the 12th. of the double of the root as often as you deſire to have a duodecimal place in the Root.

1. *Exam-*

1. Example.

I demand the ſquare root of 105032.

```
 . . .
105032
  . .
________________________
 324——01——00——05. ſquare root.
________________________
 150
  62
__________
 2632
  644
_________________
   . 56——00——00
     54——00——01
________________________________
   . 1——11——11——00—00—00—00
         04——06——00—02—00—05
____________________________________
     .——01——04——11—01—09.—11 Remain

        324
          2
        ________
    12) 648
        ________
         54——00

        324——01
               2
        ____________
    12) 648——02
        ______________
    12 ) 54——00——02
        ___________________________
          4——06——00——02——00
```

2. *Ex-*

2. *Example.*

I demand the square root of 79876—10—08—03

When there are duodecimalls in the number given take them in in the places you are to add, setting a point under the last you take, to prevent mistake.

```
 . . .
79876—10—08—03
  . .     .    .
--------------------
282—07—06 square root.
--------------------
398
 48
-----------
·1476
  562
----------------
  352—10—08
   47—00—07
--------------------
     23—06—07—03—00
      3—11—01—02—06
--------------------
```

```
     282
       2
    -------
12) 564
     47—00

     282—07
          2
    -------
12) 565—02
    -----------
12) 47—01—02
    ---------------
     3—11—01—02
    ---------------
```

This last Example appears to be a square number.

A more

A more Speedier way.

But more concise and easie, having ran through all the periods of the integers, take the 6th. of the root for the divisor to find the first duodecimal, and afterwards for divisors to find all other duodecimalls following take the * 12th. of the last divisor continually, except of the last figure therein, which you must always count so many sixes, or you may take the 6th. of the root continually. The former way gives you the reason of this, for being to multiply by 2 and divide by 12, the $\frac{1}{6}$ will produce the same, 2 being $\frac{1}{6}$ of 12.

** In taking the 12th. after the first time you need do no other than place the divisor one place back to the right hand, accounting the last figure for sixes or (which is all one) take the 6th. of it, and if you can have 6 in it count 1 in the place under it, & set the odd sixes (if any) in the next place, if there be no odd sixes, fill up the next place with two cyphers, as you may see in the Example.*

1. *Example.*

342 — 04 — 06 — 10 — 03 — 11 — 01 — 02.

18 — 06 — 00 — 06 — 01. *square Root.*

242
28

18 — 04 — 06
3 — 00 — 06

· — 01 — 06 — 10 — 03
3 — 01 — 00 — 00

1 — 06 — 10 — 03 — 11 — 01
3 — 01 — 00 — 00 — 06

· — 04 — 03 — 08 — 01 — 02 — 00
3 — 01 — 00 — 01 — 00 — 01

Remain — 1 — 02 — 08 — 00 — 01 — 11

18-06-00-06-01
18-06-00-06-01 } Multiplyed by the ratio's of 18, *viz.* ———— { 6, 3

111-00-03-00-06

333-00-09-01-06
9-03-00-03-00-06
00-09-03-00-03-00-06
01-06-06-00-06-01
1-02-08-00-01-11

342-04-06-10-03-11-01-02-00 *Proof.*

2. *Example.*

What number is that which if multiplyed into it self the product shall be 113095-07-00-08-03-06-09?

113095-07-00-08-03-06-09

336-03-06-09. *Facit.*

230
63

4195
666

199-07-00
56-00-03

31-06-03-08-03
4-08-00-06-06

3-06-00-05-03-06-09
04-08-00-07-00-09

336—03—06—09		6
336—03—06—09		
2017—09—04—06	Multiplyed by the Ratio's of 336, *viz.* by	8
16142—03—00—00		7

112995—09—00—00
84—00—10—08—03
14—00—01—09—04—06
1—09—00—02—08—00—09

113095—07—00—08—03—06—09 *Proof.*

Some Uses of Square Numbers, and their Roots.

FIrst, To make a ſquare two, three, four, or any number of times bigger than another ſquare.

Square the ſide thereof (to find the Content,) then double, treble, or quadruple the Content, as you have occaſion, and the ſquare root thereof extracted ſhews the ſide of the ſquare deſired.

So having the Diameter, Semi-diameter, or the Circumference of a Circle, to make a Circle, two, three, or four times bigger than the other.

Square it and then double, treble, or quadruple the number (as you have occaſion,) and extract the Square Root thereof, ſo you ſhall have the Diameter, Semi-diameter, or Circumference of a Circle that ſhall be two, three, or four times bigger than that other Circle.

Second-

Secondly, To find a number which multiplyed into it ſelf, the product ſhall be the number given.

The ſquare root of the number given produceth the number which multiplyed into it ſelf ſhall make up that given number. And this may be ſeen by the laſt Example.

Thirdly, To find a Geometrical mean proportion between two numbers given.

Multiply thoſe two numbers one by the other, and the ſquare root of the product is the Geometrical mean proportion between them. As ſuppoſe an Oblong were given to find the ſide of a ſquare equal, multiply the length by the breadth (to find the Content,) and the ſquare root thereof is the ſide of the ſquare equal.

Fourthly, A General of an Army having a certain number of Souldiers to know how many muſt ſtand in Rank and File, in a ſquare Battalia.

The ſquare root of the number gives the number of Souldiers that muſt ſtand in Rank and File, in a ſquare Battalia.

Fifth, The ſide of a ſquare being given, to find the length of the Diagonal.

Double the ſquare of the ſide, and the ſquare root thereof is the Diagonal.

The Cube-Root.

1. THE Extraction of the Cube Root of a number, is to find out another number, which being ſquared, (*viz.*) multiplyed by it ſelf, and then that ſquare Cubed, (*viz.*) multiplyed again by the product of the firſt multiplication, produceth the number firſt given.

Thus

Thus 1728 being a number given: 12 his Cube-Root ſquared is 144, and 144 multiplyed by 12 again (which is the Cubing of 12) makes 1728, the number given.

In like manner to find the Cube-root of 8, the root is 2, for 2 times 2 is 4, and 2 times 4 is 8.

Root.	ſquare	Cubes
1	1	1
2	4	8
3	9	27
4	16	64
5	25	125
6	36	216
7	49	343
8	64	512
9	81	729
10	100	1000

Secondly, All cube numbers are ſingle or compound, ſingle cube numbers are only thoſe 9 in the following Tablet, whoſe cube roots are expreſt by 1 figure only.

All other cube numbers above 1000, are compound cube numbers, whoſe cube-roots are expreſt by more than 1 figure. And thoſe 9 ſingle numbers in the Tablet are to be learned by heart, as the multiplication Table is.

Thirdly, All numbers are right ſingle, or compound cube numbers, whoſe roots cubed are juſt equal to the firſt given numbers.

Or elſe they be Surd-numbers, whoſe neareſt roots cubed will not be equal to the firſt numbers given, as 130, his neareſt cube-root is 5, which cubed is but 125, and 6 cubed is 216.

Fourthly, When you would find the cube-root of any number, you muſt prepare it with points, to diſtinguiſh it into cubes, or periods, by putting a point over the firſt figure, in the place of unites, and then on every 4th. figure towards the left hand.

And Note that ſo many points as the cube number hath, ſo many figures will be in the cube-root thereof.

The operation whereof mind in theſe following Examples, the precepts being very difficult to expreſſe in words, and more to underſtand.

I will

I will onely lay down this as a general Rule, and as plain as I can make it, *viz.*

First, Find the Cube of the first period, and subtract it there from (as in Division) putting the root in the quotient, and put the next period to the remain, for a new Dividend.

Secondly, Triple the root, and set the first figure under the second figure of the next period, taken down.

Then triple the square of the root, and set the first figure under the third figure of the same period, and add it together for the Divisor.

Thirdly, Seek how many times you may take it out of the Dividend, and set the figure found in the quotient, then Cube the figure found, and set the first figure under the last figure of the period, then multiply the former triple of the root by the square of the figure found, and set the first figure under the second figure of the period; then multiply the former tripple square of the root by the figure found, and set the first figure under the third figure of the period; then add it altogether and subtract it from the dividend. But here note, that if it be more than the dividend, and so may not be subtracted, you must alter the figure in the quotient, and make it less, and work it over again; but you must be sure to take alwayes as much as you can.

1. Example.

To find the Cube-root of 39304.

39304 (34
27 Cube of 3

12304 Dividend.

09 Triple of 3 the root.
27 Triple ſquare of 3 the root.

Total 279 Diviſor.

64 Cube of 4
144 Trip. of the former root 3 by the ſqu. of 4.
108 Triple ſquare of 3 multiplyed by 4.

12304 Total, which ſubtracted, from the Dividend leaves nothing, by which it appears to be a right Cube number.

Note, *For Tripling, Squaring and Cubing, you muſt work it on a waſt Paper, and ſet down their products, as in the Example.*

2. Example.

2. *Example.*

To find the Cube-root of 41063625.

```
41063625 (345
27 Cube of 3
——————
     14063 Dividend.
——————
         9 Triple of 3.
        27 Triple ſquare of 3.
——————
Total. 279 Diviſor.
——————
        64 Cube of 4
       144 Triple of 3 multiplied by the ſquare of 4.
      108      Triple ſquare of 3 multiplied by 4.
——————
Total. 12304 Which ſubtracted from the dividend.
——————
Leaves 1759625 Dividend.
——————
          102 Triple of 34.
         3468 Triple ſquare of 34.
——————
        34782 Diviſor.
——————
          125 Cube of 5.
         2550 Triple of 34 by the ſquare of 5.
        17340 Triple ſquare of 34 by 5.
——————
Total. 1759625, which ſubtracted from the laſt dividend,
                leaves nothing, whereby it appears
                to be alſo a cube number.
```

 3. *Ex-*

3. Example.

I have here sate down, onely the working, for the learners practice.

94818816 (456
64
30818
12
48
492
125
300
240
27125
3693816
135
6075
60885
216
4860
36450
3693816

456 The

	456	The Cube-root multiplyed by it
	456	self, (that is to say) squared.
	2736	
	2280	
	1824	
produceth	207936	for the square, which multiplied by
	456	the root again, that is to say, cubed,
	1247616	
	1039680	
	831744	
product	94818816	for the Cube, the same with the given number which proves the work to be true.

4. *Example.*
This ſhall be of a Surd Number.

I ſhall not ſhew you now to find the value of Fractions of compound numbers, (ſuch as the fraction here is) becauſe it will be too trouble-ſome until you can find the value of eaſier Fractions, which that you may, I will next give you two or three Examples of ſingle cube numbers, with Duodecimalls.

```
 .  .  .
202297789 (587
125
-----------
·77297
-----------
  15
 75
-----------
 765
-----------
  512
 960
600
-----------
70112
-----------
·7185789
-----------
    174
 10092
-----------
 101094
-----------
      343
    8526
 70644
-----------
 7150003
-----------
```

· · 35786 Remain, whereby it appears to be a Surd number.

To

To find the Value of a Fraction.

WHen a Fraction remains and you would know the value thereof: you muſt add three Duodecimall places to the dividend continually, taking down the Duodecimalls of your given number (if any) in their proper places, and work Duodecimally, (that is to ſay) in Trippling, Squaring and Cubing you muſt continually take the $\frac{1}{12}$ of the products as long as you have Duodecimall places to fill up: Onely obſerve that you ſtill work Duodecimally according to the ſecond way of Multiplication. As ſuppoſe you were to multiply 1 — 4 by 1 — 3 then you muſt place it as in the Margent, ſo the product will be 1 — 8 — o, whereas if you multiply it according to the firſt way (by taking the parts) it will be onely 1 8, as in the Margent: ſo the Cypher being omitted it may breed an errour in ſetting your figures.

1 — 4
1 — 3
———————
4 — o
1 — 4
———————
1 — 8 — o
———————

1 — 4
1 — 3
———————
1 — 4
o — 4
———————
1 — 8

1. *Exam-*

1. Example with Duodecimalls.

151—8—5—4 (5—4 Cube root.
Cube of 5.125 ſubtracted.

Remain -- 26—8—5—4 Dividend.

1—3—. Triple of 5.
6—3—.—. Triple ſquare of 5.

Total —— 6—4—3 Diviſor.

5—4 Cube of 4.
1—8—0—. Squ. of 4 mult. by the trip. of 5.
25—0—.—. Trip. ſqu. of 5 multiplied by 4.

Total ——26—8—5—4 ſubtracted.

.—.—.—. Remain whereby it appears to be a Cube number.

2. Exam-

2. *Example with Duodecimalls.*

      ′   ″   ‴   ⁗   ′′′′′   ′ ″
38—5—3—10—01—06 (3—4—6
27 Cube of 3.

Dividend 11—5—3—10

Divisor 2—3—9 : Trip. of 3 is 9 : trip. squ. of 3 is 2-3.

5—4 Cube of 4.
1—0—0 —— Trip. of 3 mult. by the Squ. of 4.
9—0— — — Trip. square of 3 multtiplied by 4.

10—0—5—4 Total subtracted.

Dividend. 1—4-10—6— 1—6—0 Remain

10—0 triple of 3 – 4.
2—9—4— 0 —— Trip. square of 3—4.

2—9—4—10—0 Divisor.

1—6—0 Cube of 6.
2—6—0—0 trip. of 3-4 by the sq. of 6.
1—4— 8—0—0— trip. squ. of 3-4 mult. by 6.

1—4—10—6—1—6—0 Total subtracted.

Remain — ·—·— ·—·—·—·— · whereby it appears to be also a Cube number.

3. *Exam-*

3. *Example with Duodecimalls, which shall be a Surd number.*

88-10-7-4-7-7 (4-05-6
64

24

2-00-10- 7-4 Dividend	4-5-6 Cube-root
4-01- 0— Divisor	4-5-6
10-5	17-10- 0
2-01-00	1-05·-10·
1-8-00	0-04-05-6·
	2-02- -9
1-10-01-10-5	
	19-10-06-3 squared.
·- 2-08-08-11-7-7-0 divid.	4-5-6
4-10- 7-4-3— divis.	
1-6-0	09-11-3-1-6
3-3-9-0	8-03-04-7-3-0
	79-06-01-0-0-0
2-5-3-1-6	Remain added 03-02-6-3-1
2-5-6-5-4-6-0	Cubed 88-10-07-4-7-7
Remain ——· -3-2-6-3-1-0	which proves the work to be true.

whereby it appears to be a
Surd number.

I shall now give you two Examples of compound cube numbers with duodecimalls in order to shew you how to find the value of a Fraction of such a number. One of the Examples shall be of a right cube number, and the other of a Surd.

1. *Exam-*

1. *Example.*

Let 94369-0-0-2-0-5-4 be a right compound cube number with duodecimalls, to extract the cube root thereof.

```
 .   .  '  ''  '''  ''''  '''''  ''''''        '    ''
94369—0—0—2—0—5—4 (45—06—04.
64 Cube of 4 subtracted.
Remain-30369 Dividend.
     12 - Triple of 4.
    48 —— Triple square of 4.
    492— Total.
     1 2 5—Cube of 5.
   3 0 0—- Square of 5 multiplied by the triple of 4.
  2 4 0 —— Triple square of 4 multiplyed by 5.
Total. 2 7 1 2 5—Subtracted from the last dividend.
Remain· 3 2 4 4-0-0-2 Dividend.
            11-3 — Triple of 45 Duodecimalled.
        506-03 —— Triple squ. of 45 duodecimalled.
Total —- 507-02-3 – Divisor.
            01-6-0 Cube of 6 duodecimalled. (cimalled.
        33-09-0 Squ.of 6 mult. by the trip.of 45 duode-
     3037-06 trip. squ. of 45 mult. by 6 duodecimalled.
Total - 3071-04-6-0 subtracted from the last dividend.
Remain 172-07-6-2-0-5-4 dividend.
                  11-4-6—Trip. of 45-6 duodecimalled.
     43-01-6-09-0 — Trip. squ. of 45-6 duodecim.
Total 43-01-7-08-4-6—Divisor.
                  5-4 Cube of 4 duodecimalled
        01-3-02-0-0 Squ. of 4 mult. by the trip.of45-6
  172-06-3-00-0 trip.squ.of 45-6 multiplied by 4.
total.172-07-6-02-0-5-4 subtracted.
Rem.··-··-··-··-·-· nothing, whereby it appears to
   be a Cube number.
```

2. *Ex-*

2. Example.

Let 267902—$11^{\prime}$—$11^{\prime\prime}$—$10^{\prime\prime\prime}$—$10^{\prime\prime\prime\prime}$—$3^{\prime\prime\prime\prime\prime}$—$11^{\prime\prime\prime\prime\prime\prime}$ be a compound number with duodecimalls to extract the Cube root thereof.

267902—11—11—10—10—3—11

216

·51902 (64—5—7

18

108

1098

64

288

432

46144

·5758—11—11—10

1— 4— 0

1024— 0

1025— 4— 0

·—10— 5

33— 4— 0

5120— 0

5153— 4—10— 5

605— 7—01— 5—10— 3—11

01— 4—01— 3

86— 5—04— 6—03

86— 5—05—10— 4— 3

2—4— 7

5—05— 9— 1—3

605—1— 7— 7— 9

605—7— 1— 5— 0—7— 7

Remain — ·:·—:—·—·— 9—8— 4 by which it appears to be Surd number.

Note,

Note, *In multiplying the Triple of 64 in Duodecimalls,* (viz. 1—4—0) *by the ſquare of* 5, *I multiply it by* 5, *and the product thereof by* 5 *again; ſo in multiplying the triple of* 64—5 *in Duodecimalls,* (viz. 1—4—1—3) *by the ſquare of* 7, *I multiply it by* 7, *and the product by* 7 *again. And ſo you may alwayes do in multiplying the triple by the ſquare, and may ſave much trouble.*

Some Uſes of Cubick numbers and their Roots.

THE uſe hereof is much like the uſe of the ſquares, only what they perform in the Superficies of any figure, theſe perform in the ſolid bodies thereof, for it ſerveth to find a proportion between like ſolids. Therefore,

To make any cubical or ſolid body equal to any two leſſer ones, take the ſide of each cube and multiply it cubically in it ſelf, then add them both together, and extract the cubick root from them, and that ſhall be the ſide of the cube equal to them both.

And as it falls out thus in ſquare cubical bodies, ſo is it likewiſe in round Bullets or ſolid Globes, for as the cube of the Diameter of one Bullet, is to its weight, ſo is the cube of another Diameter to its weight.

So as the cube of the Diameter of one ſolid Globe is to its Solid Content, ſo is the cube of the Diameter of any other ſolid Globe, to its Solid Content.

And as it is in round and ſquare ſolid bodies, ſo it is in all other ſolid bodies. So that knowing the Mold and burthen of one Ship, you may build another thereby on the ſame mold of what burden you pleaſe, after this manner, *viz.* Take all the dimenſions that make the ſhape and mold of the ſhip, and cube them ſeverally, and ſo make each part proportionable, &c.

The

The Rule of Three.

HEre you have three numbers given to find a fourth. Where obſerve.

If more requires more, or leſs requires leſs, the proportion is Direct. And then you muſt multiply the ſecond number by the third, or the third by the ſecond, and divide the product by the firſt. But

If more requires leſs, or leſs requires more, the proportion is Reverſe; and then you muſt multiply the ſecond number by the firſt, or the firſt by the ſecond, and divide the product by the third. And the Quotient in both gives your deſire.

The firſt and third numbers muſt ſtill be of the ſame kind or ſpecies, and the anſwer will always be of the ſame kind or ſpecies with the ſecond number.

Three or four Examples in this Rule will be ſufficient, there being but little uſe thereof in this Arithmetick.

1. *Example.*

By the Area of one Circle to find the Content of another, knowing the diameters of both.

As the ſquare of the Diameter of the known Circle, is to its Area, ſo will the ſquare of the diameter of the unknown Circle be to its Area.

The Area of a Circle whoſe diameter is 2 foot 4 inches is 4 foot 3 inches 4 parts, I demand the Area of another Circle whoſe diameter is 2 foot 11 inches?

Foot.

Foot. Inch.	*Foot. Inch. Parts.*	*Foot. Inch.*
2—4	4 — 3 — 4 : :	2 — 11
2—4	8 — 6 — 1	2 — 11
4—8	34 — 2 — 8	2 — 8 — 01
0—9—4	2 — 1 — 8	5 — 10
	0 — 4-3-4	
5—5—4		8 — 06 — 01
5—5—4)	36 — 4 — 8-3-4	(6 — 8 — 2 *fac.*
	36 — 4 — 8	that is to say 6 foot 8
	32 — 8 — 0	inches and 2 parts.
	·3 — 8 — 8 — 3	
	3 — 7 — 6 — 8	
	· — 1 — 1 — 7 — 4	
	10 — 10 — 8	
	· — 2 — 8 — 2	

2. *Example.*

By the diameter of a Circle to find the Circumference. As 7 to 22, so is the diameter of any Circle to its Circumference.

I demand the Circumference of a Circle whose diameter is 3 foot.

Inches.	Foot.	Inches.	Foot.
0 — 7 —	1 —	10 : : —	3

3 foot. inches.

0 — 7) 5 — 6 (9 — 5 *Facit.*
5 — 3

· — 3 — 0
2 — 11

: — 01

3. *Example*

3. *Example.*

By the Circumference to find the diameter. As the Circumference of one Circle is to its diameter, so is the Circumference of another Circle to its diameter.

foot. inches.
The Circumference of a Circle is 9 — 5. whose diameter is 3 foot. I demand the diameter of another Circle, whose Circumference is 1 foot 10 inches.

```
Foot. inches. foot.     foot.  inches.
9——5——3 : :              1——10
                              3
                   ——————————— Foot. Inches
           9——5) 5——06— (0 —— 7 Facit.
                 ——————————
                 5——06—0
                 5——05-11
                 ——————————
                 ·——· ·-01
```

4. *Example.*

To find the solid Content of a Globe by the diameter.

As 21 is to 11 so is the cube of the Diameter to its solid Content.

Foot. inches.
There is a Globe whose diameter is 1 —— 9, I demand its solid Content.

Foot.

```
fo. inch.            foot. inches.
1—9—0—11 : : —1——9——0
               1——3——9
             ───────────────
               3——0——9
               2——3——6—9
             ───────────────
               5——4——3—9
                       0—11
             ────────────────────
      1—9) 4——10——11—5—3(2-9-8-3 fac.
               3——6
             ───────────────────
               1——4——11
               1——3——9
             ─────────────────────
               ·——1——2——5
                  1——2——0
                ─────────────────────
                  :——·——5——3
                        5——3
                     ──────────
                        ·——·
```

5. *Example.*

By the ſolid Content of a Globe, to find the diameter.

As 11 is to 21, ſo is the ſolid Content to a fourth number, the Cube root whereof will be the diameter.

There is a Globe whoſe ſolid Content is 2—9—8—3, (foot.) I demand its Diameter.

```
                 11——21 : : ——2——9——8——3
                           ──────────────────
I multiply by the Ra-       19——7——9——9
tio's of 21, viz, 7 & 3.   ──────────────────
                        11(58 --11——5——3
                           ──────────────────
                            5——4——3——9
                           ──────────────────
             Facit.  1——9——·——· cube root.
```

Duodecimal Mensuration:

BOOK. II.

Shewing how to Measure Superficies, and Solids.

I. *Of Superficies.*

First, *Of Squares, and Oblongs or Parallelograms.*

UNder which Head are contained the measuring of Board, Glass, Pavement, Tyling, Wainscot, Walls, and the like; whose forms are either Squares or Parallelograms.

Note, *If the Carpenters Rule be made* 3 *foot long with a Joynt at each foot, it may be folded to* 1 *foot, to be carryed in the Pocket. And if these following Lines be put thereon, and Duodecimally divided, all forms usual may be measured and cast by one and the same Rule,* viz. *Multiplication in Duodecimalls; and that whether they are to be measured by the square or Superficiall foot, or by the Solid foot, or by the square yard,* &c.

The Lines.

foot. inches.

1. The square foot ———— ———— 1—00—00—00—00.
2. The diameter square foot ——— 1—01—06—05--5-5.
3. The diameter squ. inscribed foot.-0—08—04—09—8 -
4. The circumfer. squ. inscrib. foot—4—05—10—5—
5. The circumfer. squ. diameter foot. 3—6—6—6—10—2.
6. The square yard.— ———— —— 3—0—0—0—0—

And having fully instructed you in the Aliquot parts of 12, I shall now shew you their excellent use in measuring.

First, Of Boards and Glass, both being measured by the foot square.

Problem 1.

A peice of Board 16 inches square, how many square feet doth it contain.

The Rule.

Multiply the length by the breadth, which may be performed (as you have been taught) by the Aliquot parts of 12, that is, to say 1 foot 4 inches by 1

foot. inches.
1 — 04
05 — 04
———— ——— —— ——
facit.—1 —— 09 —— 04
——— ———— ————

foot 4 inches, as in the Margent.

Pro-

Problem 2.

A Board 5 foot long, and 10 inches broad, how many ſquare feet doth it contain?

Foot.	inches.
5 —	00
2 —	06
1 —	08
Facit. 4 —	02

Or thus,
inches.

10
5
4 — 02

Problem 3.

A Board 16 $\frac{1}{2}$ foot long, and 9 $\frac{1}{2}$ inches broad, how many ſquare feet doth it contain?

Foot.	inches.	
16 —	06	
8 —	03	
4 —	01 —	06
	08 —	03
Facit 13 —	00 —	09

Note, I take $\frac{1}{2}$ the given number for 6 inches, $\frac{1}{2}$ of that for 3 inches, and $\frac{1}{2}$ the given number again ſetting it one place back for 6 parts, according to the 5th. Rule in Multiplication.

Problem 4.

A Board 16 $\frac{3}{4}$ foot long and 1 foot 3 inches and $\frac{1}{2}$ broad, how many ſquare feet are there contained in it?

Foot.	inches.	Parts.	
16	09	00	
4	02	03	
	08	04	06
Facit 21	07	07	06

I take the whole given number for 1 foot, the $\frac{1}{4}$ of the given number for 3 inches, and the $\frac{1}{2}$ of the given number placed one place back to the right hand for $\frac{1}{2}$ an inch.

Problem 5.

20 Boards each 16 foot, 6 inches long, and 1 foot 2 $\frac{3}{4}$ inches broad, how many ſquare feet do they contain?

20. Boards

160

320
10

330
55
13 — 9
6 — 10 — 06

Facit — 405 — 07 — 06

Here I multiply by 8, and the product by 2, becauſe 2 times 8 is 16, according to the 8th. Rule in Multiplication, (8 and 2 being the ratio's of 16,) then I take the $\frac{1}{2}$ for 6 inches, and add the two products together, then I take the whole total for 1 foot of breadth, and for 2 inches I take the $\frac{1}{6}$, and for 6 parts the $\frac{1}{4}$ of that, and for 3 parts the $\frac{1}{2}$ of that.

Prob.

Problem 6.

foot. inches.

100 Boards each 15 —— 04 long, and 1 foot 5 inches broad, how many ſquare feet do they contain?

100 Boards.

500

1500
33 —— 04

1533 —— 04
511 —— 01 —— 04
127 —— 09 —— 04

Facit—2172 —— 02 —— 08

Here again, I multiply the 100 Boards by the ratio's of 15, *viz.* 3 and 5, then I take the $\frac{1}{3}$ of the Boards for 4 inches, and add the two products together. Then for the breadth, I take the whole total for 1 foot, and for 4 inches I take the $\frac{1}{3}$, and for 1 inch the $\frac{1}{4}$ of that,and add the 3 products together.

Problem 7.

A Board 7 primes or inches, and 2 ſeconds or parts long, 3 primes 6 ſeconds broad (that is to ſay, 3 inches and $\frac{1}{2}$ broad,)how many long inches of the ſquare foot contains it? Or if you will (and which is all one) how many primes of the ſquare foot contains it?

primes ſeconds.

7 —— 02.

1 —— 09 —— 06
03 —— 07

Facit—2 —— 01 —— 01

Facit 2 inches or primes, 1 ſecond, 1 third broad, and 12 inches or primes long.I take $\frac{1}{4}$ for 3 inches or primes, and then $\frac{1}{2}$ for 6 ſeconds placed one place back,by the 5th. Rule in multiplication.

Problem 8.

A Board 14 foot 6 primes long, and 9 primes broad at one end, and 7 primes broad at the other, how many ſquare feet doth it contain?

foot.	inches.		
14 —	06 the length		9
			7
4 —	10		16
4 —	10		
			8 true breadth.
9 —	08 *Facit*, I take —		
	$\frac{2}{3}$ for 8 inches or primes.		

Problem 9.

Having the Breadth to find how much in length will make the foot ſquare?

The Rule.

As the breadth is to 1 ſo is the ſquare foot to the length, that ſhall make the foot ſquare, whether in feet or inches: therefore divide the ſquare foot by the breadth given?

1. *Example.*

A Board 9 inches or primes broad; what length makes the ſquare foot?

foot inches
9) 12 (1 — 04
3 *Facit.*

Note, 12 long inches of the ſquare foot are each 12 inches long, and length being demanded, it muſt be 1 foot 4 inch. for the quote.

2. *Ex-*

2. Example.

A Board 10 inches $\frac{3}{4}$ broad, what length makes the ſquare foot ?

```
10——9) 12——00 (1–01–04–08 facit the length
       10——09                of the ſqu.foot.
   ————————————
        1——03——00
   ———————————————
        ·——04——03——00
   ——————————————————————
             ·——08——00——00
         ————————————————————
                  ·——10——00
              ————————————————
```

Problem 10.

There is a Window that hath 4 panes of Glaſs, each pane 1 foot 9 inches and $\frac{1}{2}$ broad, and 4 foot 7 $\frac{1}{2}$ inches long, how many ſquare feet of Glaſs are there in all ?

```
         Foot.
          4——07——06
        ——————————————
         18——06——00
          9——03——00
          4——07——06
             09——03
        ——————————————
  Facit.33——01——09
        ——————————————
```

I multiply the length by 4 the number of panes, *facit* 18 foot, 06 inches, then I take 18 foot 06 inches for 1 foot breadth, the $\frac{1}{2}$ for 6 inches, the $\frac{1}{2}$ of that for 3 inches, and the $\frac{1}{2}$ of 18 foot, 06 inches again for 6 parts, ſetting it one place back. Or the 6th. of 3 inches.

Prob.

Problem 11.

There is a Window that hath 5 panes of Glaſs, each pane being 3 foot, 10 inches, 9 parts high, and 15 inches 10 parts broad, how many ſquare feet contains it?

Foot.	inches.	parts.		
3	10	09 high.		
19	05	09		
4	10	05	03	
	9	08	10	06
	6	05	11	00
Facit.–25	08	05	00	06

I multiply the heighth by the 5 panes, and the product by the breadth as formerly.

Thus far concerning Foot Meaſure.

Second-

Secondly, *Of Pavements, VVainſcot, Tyling, Plaiſtering,* &c. *Theſe being meaſured by the Yard ſquare.*

Problem 1.

THere is a Court to be paved that is 18 $\frac{1}{2}$ foot long, and 14 $\frac{1}{2}$ foot broad, how many yards ſquare will there be in the Pavement?

Firſt, I put the feet into yards both of the length and breadth, and then multiply the one by the other; thus, *viz.* Firſt I multiply 6 yards, 0 foot, 6 inches, by 4 yards, then for the 2 foot 6 inches, I take the $\frac{1}{3}$ for 1 foot (becauſe 1 foot is the $\frac{1}{3}$ of a yard,) then for 1 $\frac{1}{2}$ foot, I take the $\frac{1}{2}$, that being the $\frac{1}{2}$ of a yard.

Note, *If you have a yard Duodecimally divided, you may meaſure all Superficies by it that are to be meaſured by the yard ſquare and work as you are taught for the foot ſquare.*

yards.	foot.	inches.	
6	0	06	
4	2	06	
24	2	00	
2	0	02	
3	0	03	
29	2	05	*Facit.*

Or

Or thus,

yards.	foot.	inches.
4	2	06
6	0	06
29	0	00
	2	05
29	2	05 *facit.*

Here I multiply, 4 yards, 2 foot, 06 inches, firſt by 6 yards, *facit* 29 yards, then for 6 inches, I take the $\frac{1}{6}$ becauſe 6 inches is the $\frac{1}{6}$ of a yard.

The Common way.

Foot.	inches.	
18	06	
14	06	
129	06	by the ratio's of 14.
259	00	
9	03	
9)268	03	

Note, *The Common way is to multiply the length by the breadth in feet and inches, and to divide the product by 9, becauſe there are 9 ſquare feet in a ſquare yard, and accordingly I have wrought it by the common way, that you may ſee the verity of the Duodecimal way.*

29 — 02 — 05 *facit.* In dividing by 9 there remained 7 at feet which muſt be accounted $\frac{7}{9}$ yard, therefore 7 multiplied by 3 *facit* 21, which divided by 9 *facit* 2 foot, and 3 remains which are $\frac{3}{9}$ of a foot, *viz.* 4 inches, and the 3 inches ſquared makes 9, which divided by 9 makes 1 inch, which added to 4 *facit* 5.

Problem 2.

A Room 18 foot long, and 15 foot broad is to be paved with Stones of 8 inches long, and 6 inches broad, how many ſuch ſtones will pave it?

Foot.

Foot.		inches.	
18		8	in multiplying 8 inches
15		6	I take the $\frac{1}{2}$ *facit* 4 ſqu. Inches.
90	by the ratio's (of 15.	4	inches or $\frac{1}{3}$.
270			
3	to divide by 4 inches is multiplying by 3.		
810	*Facit.*		

Problem 3.

There is a piece of Wainſcot that is 16 $\frac{1}{2}$ foot long, and 8 $\frac{3}{4}$ foot broad, how many ſquare yards doth it contain?

	yards.	Foot.	Inches.	
	5	1	06	
	2	2	09	
	11	0	00	
	1	2	06	
	2	2	03	
		1	04	6
Facit.	16	0	01	6

Or thus.

yards.	Foot.	Inches.		
2	2	09		
5	1	06		
14	1	09		
1	1	04	06	
16	0	01	06	*Facit*, the (ſame again.

Prob.

Or thus,

yards.	foot.	inches.
4	2	06
6	0	06
29	0	00
	2	05
29	2	05 *facit.*

Here I multiply, 4 yards, 2 foot, 06 inches, firſt by 6 yards, *facit* 29 yards, then for 6 inches, I take the $\frac{1}{6}$ becauſe 6 inches is the $\frac{1}{6}$ of a yard.

The Common way.

Foot.	inches.	
18	06	
14	06	
129	06	by the ratio's of 14.
259	00	
9	03	
9)268	03	

Note, *The Common way is to multiply the length by the breadth in feet and inches, and to divide the product by 9, becauſe there are 9 ſquare feet in a ſquare yard, and accordingly I have wrought it by the common way, that you may ſee the verity of the Duodecimal way.*

29 — 02 — 05 *facit.* In dividing by 9 there remained 7 at feet which muſt be accounted $\frac{7}{9}$ yard, therefore 7 multiplied by 3 *facit* 21, which divided by 9 *facit* 2 foot, and 3 remains which are $\frac{3}{9}$ of a foot, *viz.* 4 inches, and the 3 inches ſquared makes 9, which divided by 9 makes 1 inch, which added to 4 *facit* 5.

Problem 2.

A Room 18 foot long, and 15 foot broad is to be paved with Stones of 8 inches long, and 6 inches broad, how many ſuch ſtones will pave it?

Foot.

Foot.		inches.	
18		8	in multiplying 8 inches
15		6	I take the $\frac{1}{2}$ *facit* 4 ſqu.
			Inches.
90	by the ratio's	4 inches or $\frac{1}{3}$.	
270	(of 15.		
3	to divide by 4 inches is multiplying by 3.		
810 *Facit.*			

Problem 3.

There is a piece of Wainſcot that is 16 $\frac{1}{2}$ foot long, and 8 $\frac{3}{4}$ foot broad, how many ſquare yards doth it contain?

	yards.	Foot.	Inches.	
	5	1	06	
	2	2	09	
	11	0	00	
	1	2	06	
	2	2	03	
		1	04	6
Facit.	16	0	01	6

Or thus.

yards.	Foot.	Inches.		
2	2	09		
5	1	06		
14	1	09		
1	1	04	06	
16	0	01	06	*Facit*, the (ſame again.

Prob.

Problem 4.

There is a room to be Plaiſtered, the two ſide-walls being 14 foot long, and the two end-walls 10 foot broad, and the heighth of the room 7 $\frac{1}{2}$ foot, *Q.* How many yards are there in the Plaiſtering of the ſame in the walls and over the Head.

	7	6		heighth of the Walls.
	7	6		
	10	0		breadth of the room.
3)	25	0		
yards	8	1		foot the whole breadth.
	4	2		the length.
	33	1		
	2	2	04	
	2	2	04	
	38	2	08	product of the
	16	2	00	breadth by the length.
	55	1	08	*Facit.*

yards.

yards	foot.	
3	01	breadth of the end walls.
5	00	length of both end walls.
16	02	prod. of breadth by length. which added with the former product, as you ſee it done you'l find

	yards.	foot.	inches.	
There are	55	1	08	of Plaiſtering in the Room.

Note, *Workmen include Windows, Chimneys and Doors in the length and heighth of the Walls, in regard of Plaiſtering their ſides,* &c.

Note again, *I have met with an Errour too commonly uſed by ſome Plaiſterers in the Countrey, which is this,* viz. *They uſually agree for the Lathing and Plaiſtering of Rooms for ſo much the yard ſquare, and when they come to meaſure a Room ſo Lathed and Plaiſtered, they take in the heighth of the two end Walls with that over head for the length, and the heighth of the two ſide Walls with that over head again for the breadth, and multiply one by the other, whereby they make it* $\frac{4}{9}$ *more than the truth : And though* I *have demonſtrated it to them thus,* viz. *That if the* 4 *Walls with that over head were preſt down on a plane (ſuppoſing the room to be ſquare) it would make a perfect croſs. And multiplying the Croſs, as if it were a perfect ſquare, they take in the four vacancies that make up the ſquare into the work, yet they would not be ſatisfied: ſo difficult is it to convince a man of a profitable Errour ; whereas the heighth of the ſide walls muſt be added with the breadth of the Room for the true breadth, and that muſt be multiplyed by the Rooms length, and then the two end walls muſt be meaſured by themſelves, and added with the former product, as in the foregoing Problem.*

Third-

Thirdly, *Of Walls.*

THeſe being meaſured by the Rodd of 18 foot ſquare, by which in ſome places Tyling is meaſured alſo, and in ſome places by the yard ſquare. And in ſome places Walls are meaſured by the Perch or Rodd of 16 ½ foot ſquare. In ſome places by the Perch of 16 foot long, and 1 foot high.

Here needs no Example of meaſuring by the yard ſquare, having given you Examples ſufficient for that already.

Problem 1.

There is a Wall 54 foot long, and 27 foot high, how many ſquare Perch is it, at 18 foot to the perch.

Reſolution.

Multiply ½ the length by ½ the heighth, and take the 9th. of the product, and the 9th. of that.

27
13 —— ½

81
270
13 —— ½

364 —— ½

40 —— ½

facit.—4 —— ½

Note, *In taking the* 9*th. the firſt time there remained* 4, *which I put into halfs, ſaying* 2 *times* 4 *is* 8, *and the odd* ½ *put to it made* 9, *then I took the* 9*th. of that which was* ½ *again. Then in taking the* 9*th. the ſecond time it happened in like manner, and I did the like again.*

Prob. 2.

Prob. 2.

There is a Wall 54 foot long, and 27 foot high, how many ſquare perch contains it at 17 $\frac{1}{2}$ foot?

Reſolution.

Multiply the length by the highth, and the product by 4, then take the 7th. of that product, the 7th. of that, and the 5th. of that, and the 5th. of that again.

$$\begin{array}{r} 54 \\ 27 \\ \hline 378 \\ 108 \\ \hline 1458 \\ 4 \\ \hline 5832 \\ \hline 833 - \frac{1}{7} \\ \hline 119 - 0 - \frac{1}{7} \\ \hline 23 - \frac{5}{7} - \frac{4}{7} - \frac{2}{5} \\ \hline 4 - \frac{5}{7} - \frac{2}{7} - \frac{1}{5} - \frac{2}{5} \\ \hline \end{array}$$

Perch.

Facit—4—$\frac{5}{7}$—$\frac{1}{7}$—$\frac{1}{5}$—$\frac{3}{5}$, that is to ſay 4 perch and $\frac{3}{4}$ and ſome what more.

Problem 3.

There is a Wall 54 foot long: and 27 foot 8 inches high, how many perch contains it at 16 foot long, and 1 foot high.

```
   54
   27——8
——————————
  378
 1080
   18
   18
——————————
 1494
```

divided by the ratio's of 16, *viz.* 4 and 4.

```
  373——8
——————————
   93——6 Facit.
```

Problem 4.

How many Bricks will raise a Wall of a Brick and ½ thick, 20 foot long, and 18 foot high, when 15 Bricks will raise 1 foot square of that thickness?

Resolution.

Multiply the length by the highth (to find the square feet contained in all,) and then multiply the product by the number of Bricks that will raise 1 foot square.

```
    18
    20
——————————
   360
    15
——————————
  1800
  360
——————————
Facit 5400 Bricks.
```

1. *Exam-*

Note, *In* Smirna *Turky Carpets are meaſured and ſold by the yard ſquare, the yard being 3 Engliſh feet, and accordingly I have given theſe two Examples.*

1. *Example.*

A Carpet that meaſures 4 yards 2 foot in length and 2 yards 2 foot in breadth, how many yard doth it contain?

```
yards   foot.
  4 —— 2 long
  2 —— 2 broad
————————————
  9 —— 1
  1 —— 1 —— 08
  1 —— 1 —— 08
————————————————
 12 —— 1 —— 04 Facit.
————————————————
```

Here I multiply firſt 4 yards 2 foot by 2 yards, *facit* 9 yards 1 foot; then to multiply 4 yards 2 foot, by 2 foot, I ſay 2 foot is $\frac{2}{3}$ of a yard, therefore I take $\frac{1}{3}$, and ſet it down twice.

2. *Example.*

A Carpet 4 yards 2 foot and 2 inches long, and 2 yards 1 foot and 2 inches broad, how many ſquare yards contains it.

```
yards  foot  inches.
  4 —— 2 —— 2 long
  2 —— 1 —— 2 broad
————————————————————
  9 —— 1 —— 4
  1 —— 1 —— 8 —— 8
       0 —— 9 —— 5 —— 04
——————————————————————————
 11 —— 0 —— 10 —— 1 —— 04 Facit.
——————————————————————————
```

Here I multiply firſt by 2 yards then I take the $\frac{1}{3}$ for 1 foot, and the $\frac{1}{6}$ of that for 2 inches.

Thus far concerning Squares and Oblongs the uſual formes of Boards, Glaſs, Wainſcot, Tyling, &c.

Secondly, *Of Triangles.*

HAving given you Rules and Examples ſufficient to find the contents or Areas of Squares, and Oblongs, I ſhall now ſhew you how you may find the Contents or Areas of Triangles.

A Triangle is a figure contained by 3 ſides either equal or unequal, which are right Angled, or obtuſe, or acute, or equilateral, of 3 ſides all equal, but all are meaſured by this general Rule, *viz.*

Multiply the whole Baſe by $\frac{1}{2}$ the perpendicular, or $\frac{1}{2}$ the Baſe by the whole perpendicular.

Problem 1.

There is a Triangle whoſe Baſe is 5 foot 6 inches, and the perpendicular 4 foot 3 inches. *Q.* How many ſquare feet doth it contain?

	foot.	inches.	
half the Baſe	2 —	09	
the perpendicular	4 —	03	
	11 —	00	
	0 —	08 —	03
	11 —	08 —	03 *Facit.*

Problem 2.

There is a Triangle whoſe baſe is 10 foot 8 inches, and the perpendicular 8 foot 10 inches. *Q.* How many ſquare feet doth it contain?

foot.

	foot.	inches.	
The Baſe	10	08	
½ the perpend.	4	05	
	42	08	
	3	06	08
	0	10	08
Facit.	47	01	04

Note, If you cannot come to meaſure the perpendicular, then you may find the Content by the 3 ſides onely, Thus, *viz.*

1. Add the 3 ſides together to get the ſumme and ½ that for the ½ ſumme of the ſides, called the Perimeter, and Semi-perimeter.

2. From the Semi-perimeter ſubtract each ſide ſeverally to get the 3 differences.

3. Multiply one difference by another, and that product by the third difference, and again that product by the Semi-perimeter; after the manner of continual Multiplication.

4. The ſquare Root of the laſt product is the ſuperficial Content of the Triangle.

Example. foot. inches.

A Triangle whoſe ſides are, *viz.* { 8—10, 10—08, 9—06 } I demand its ſuperficial Content.

Perimeter	29—00 Total.
Semi-perimeter	14—06
firſt difference	5 - 08
ſecond difference	3—10
third difference	5—00

 foot.

```
Foot
  5 —— 08 firſt Difference by
  3 —— 10 the ſecond Difference.
 ——————————————————
 17 —— 00
  2 —— 10
  1 —— 10 —— 08
 ——————————————————
 21 —— 08 —— 08 Product.
              5 by the third Difference.
 ——————————————————
108 —— 07 —— 04 Product, by
 14 —— 06 ——    the Semy-Perimeter.
 ——————————————————
760 —— 03 —— 04
 ——————————————————
1520 —— 06 —— 08
  54 —— 03 —— 08
 ——————————————————
1574 —— 10 —— 04 Product.
 ——————————————————
  39 —— 08 —— 02 Square Root.
 ——————————————————
 674
  69
 ——————————————————
  53 —— 10 —— 04
   6 —— 06 —— 08
 ——————————————————————————————
   1 —— 05 —— 00 —— 00 —— 00
        06 —— 07 —— 04 —— 02
 ——————————————————————————————
     —— 03 —— 09 —— 03 —— 08 Remain.
```

So that the Content of the Triangle is 39 Foot, 8 Inches, 02 Parts, and ſomewhat more, which is very inconſiderable.

Note again; To find the Superficial Content of an Equilateral Triangle without the Perpendicular, you may either 1. Add the 3 Squares made of the Semy-ſides together, or

or Multiply the Square of the Semy-ſide by 3, which is all one, and take the Square Root of the Total or Product for the Superficial Content.

2. Or you may Multiply the Square of one of the Semy-ſides by the Square Root of 3, which is 1 : 8 : 9 : 4 : 11 : + and it will produce near the ſame.

Example.

An Equilateral Triangle whoſe ſide is 13 Foot—4 Inches, what is its Superficial Content ?

By the firſt way.

```
Foot.  Inches.
  6 —— 08  the Semy-ſide.
  6 —— 68
―――――――――――――――
 40 —— 00
  2 —— 02 —— 08
  2 —— 02 —— 08
―――――――――――――――
 44 —— 05 —— 04  Square of the Semy-ſide.
              3
―――――――――――――――
133 —— 04 —— 00
―――――――――――――――
Sq. Root 11 —— 06 —— 06  Facit.
―――――――――――――――
 ·33
  21
―――――――――――――――
 12 —— 04 —— 00
  1 —— 10 —— 06
―――――――――――――――――――――――――――
  1 —— 01 —— 00 —— 00 —— 00
       01 —— 11 —— 00 —— 06
―――――――――――――――――――――――――――
  · —— 01 —— 05 —— 09 —— 00
```

By the ſecond way.

	1 —	8 —	9 —	5		The Square Root of 3.
				6 —	8	
	10 —	4 —	08 —	6		
		10 —	04 —	08.		
		3 —	05 —	05 —	8	
Facit	11 —	06 —	06 —	07 —	8	near the ſame again.

Problem 3.

If you deſire to find the Perpendicular of a Triangle by Arithmetick. the Rule is, *viz.*

1. Square every ſide ſeverally.
2. Then add the Square of the Baſe or longeſt ſide to the Square of the ſhorteſt ſide.
3. And from the ſum Subtract the Square of the other ſide; halfing the reſidue.
4. Divide half the reſidue or Remainder by the Baſe, and Square the Quotient.
5. Subtract this Square from the Square of the leaſt ſide firſt added to the Square of the Baſe, noting the Remainder.
6. The Square Root of this Remainder is the perpendicular required.

Exam-

Example.

A Triangle whose Base is 2 foot, and the sides are each 1 Foot 6 Inches, I demand the length of the perpendicular.

```
      Foot.     Foot.
Base  2—00—4—00 Squared.
     {1—06—2—03
Sides{     ————
     {1—06—2—03 Square of the lesser side}
            4—00 Square of the Base      }Added.
          —— ——
            6—03 Total.
            2—03 Sq. of the other side subtracted.
        —— ——
            4—00 Remain.
        —— ——
        2) 2—00 The ½ of the remain,div.by the Base.
        —— ——
            1—00 Quotient, which squared makes (1)
        — — —— subtra. from the Sq.of the least side.
            1—03 Remain.To find the Sq.root thereof.
        —— — ——
  Facit 1—01—04 Square Root.
        —— — ——
        ·—03—00
            2—01
  —— —— —— ——
          ·—11—00—00
              2—02—04
      —— —— ——
              2—02—08
```

Here

Here Note two Things more.

First, To find the Base of a right angled Triangle, both sides being given, accounting the longest side the Base, Add the squares of the two sides given together, and the square Root of the Total is the other side.

Secondly, To find a side having the Base, and the other side given.

Take the square of the side out of the square of the Base, and the square Root of the remain shall be the side required.

Problem 4.

There is a poining end Wall of a House 26 foot long, and 18 foot high to the Roof, and the perpendicular of the Triangle from the Ridge of the House to the Base of the Triangle is 12 foot; How many square Rods doth this poining end Wall contain, of 18 foot to the Rod?

13		13
9		3
117		39
39		
156		
17 —	4	
Facit 1 —	11 —	$\frac{1}{9}$

I Multiply $\frac{1}{2}$ the length by $\frac{1}{2}$ the highth of the Oblong of the Wall, then I Multiply $\frac{1}{2}$ the Base of the Triangle by $\frac{1}{2}$ of $\frac{1}{2}$ of the perpendicular, and add the two Products together, then I take the 9th of the Total, and the 9th of that, (according to the first Problem of Walls,) and it maketh 1 Rod—11—$\frac{1}{9}$ which is near 2 Rods.

Problem

Problem 5.

If the ſame poining end Wall were to be meaſured by a Rod of 17 $\frac{1}{2}$ foot, how many ſuch ſquare Rods would it contain?

26	26
18	6
208	156
26	
468	
156	
624	
4	
2496	
$356 — \frac{4}{7}$	
$50 — \frac{6}{7} — \frac{4}{7}$	
$10 — \frac{1}{7} — \frac{2}{7} — \frac{1}{5}$	
Facit $2 — 0 — \frac{1}{7} — \frac{2}{5} — \frac{1}{5}$	

Here I Multiply the whole Area by 4, and then I take the 7th of the Product, the 7th of that, the 5th of that, and the 5th of that again, (according to the ſecond Problem of Walls,) and I find it 2 Rods, and ſomewhat more.

Problem 6.

If a Triangle or Gable were to be riſen on a ſquare Wall of 26 foot long with a ſingle Brick, and the perpendicular of the Triangle 18 foot, how many Bricks will ſerve to raiſe it, when 10 Bricks will ſerve to raiſe 1 foot ſquare?

18 Perpendicular.

26 Baſe. 9 The half therefore.
9

234 ſquare feet in all.
10

Facit . 2350 Bricks will raiſe it.

A Queſtion for Practice of what hath been Taught.

THere is a Wall 12 foot long, and 8 foot high to be hung or covered with green Bayes that is $\frac{3}{4}$ of a yard broad, how many yards of Bayes will perform it?

Note, Before you can bring this queſtion to the Rule, you muſt ſquare the Wall, *viz.* multiply the length by the highth in feet, and the product gives the number of ſquare feet contained in it; then becauſe the Queſtion referrs to yards, you muſt bring thoſe ſquare feet into ſquare yards, by dividing by 9 becauſe 9 ſquare feet are contained in one yard ſquare?

Operation.

12
8

9)96

10 —— $\frac{6}{9}$ or $\frac{2}{3}$ ſquare yards in the Wall. Then ſay,

If

If $\frac{3}{4}$ of a ſquare yard require 1 yard in length, how many yards in length of the ſame breadth will 10 $\frac{2}{3}$ yards ſquare require.

$\frac{3}{4}$ — 1 yard — 10 $\frac{2}{3}$ yards
4
3) 42 $\frac{2}{3}$
Facit. 14 : 0 $\frac{2}{9}$ yards

Or you may do it by two Operations, Thus: ſay *viz.*

If 3 require 1 what will 4 require.

Facit. 1 — $\frac{1}{3}$ yard, Then

If 1 yard ſqu. require 1 — $\frac{1}{3}$ yard what will 10 : $\frac{2}{3}$ yards re-
3 : $\frac{1}{3}$ $\frac{2}{3}$ (quire

Facit. 14 — 0 — $\frac{2}{9}$ the ſame again.

Thirdly, *Of Circles.*

Problem 1.

TO find the Circumference by the Diameter.

The Analogy is,

As 1 is to 3 $\frac{1}{7}$ ſo is the Diameter to the Circumference, 3 $\frac{1}{7}$ being the Circumference of a Circle whoſe Diameter is 1.

Exam-

Example.

A Circle whoſe Diameter is 4 foot 5 inches, I demand the Circumference?

foot	inches	
4 ——	05	
	3 ——	$\frac{1}{7}$
13 ——	03	
	07 ——	$\frac{4}{7}$
13 ——	10 ——	$\frac{4}{7}$ Circumference.

foot. inches.
Facit—13 —— 10 —— $\frac{4}{7}$. The $\frac{4}{7}$ being ſomewhat above $\frac{1}{2}$ an inch.

Problem 2.
To find the Diameter by the Circumference.

The Analogy.
As 1 is to $\frac{7}{22}$ ſo is the Circumference to the Diameter, $\frac{7}{22}$ being the Diameter of a Circle whoſe Circumference is 1.

Example.

foot. inches.
A Circle whoſe Circumference is 13 —— 10 —— $\frac{4}{7}$, I demand the Diameter?

Foot.	inches.	
13 ——	10 ——	$\frac{4}{7}$
6 ——	11 ——	$\frac{2}{7}$
	07 ——	$\frac{4}{7}$
		4
2 ——	06 ——	$\frac{2}{7}$
4 ——	05 ——	0 *facit*

Note here, $\frac{11}{22}$ is $\frac{1}{2}$: and $\frac{7}{22}$ is $\frac{4}{11}$ leſs then the $\frac{1}{2}$, therefore I ſubtract $\frac{4}{11}$ of the Semi-circumference from the Semi-circumference, and it gives 4 foot 5 inches, being the contrary of the former Problem.

Or,

Or which is yet better, Multiply the Semi-circumference by 7, and take the 11th. of the product, and it gives the Diameter, for the Diameter is $\frac{7}{11}$ of the Semi-circumference.

$$
\begin{array}{cl}
 & 6 \text{ — } 11 \text{ — } \tfrac{3}{7} \\
 & \qquad\qquad\quad 7 \\
\hline
\tfrac{1}{11} & 48 \text{ — } 7 \text{ — } 0 \\
\hline
 & 4 \text{ — } 5 \text{ — } 0 \text{ the ſame again.} \\
\hline
\end{array}
$$

Problem 3.

To find the Superficial Content by the Diameter.

Note, every Circle is $\frac{11}{14}$ of the ſquare made of his Diameter, or $\frac{11}{14}$ is the Area of a Circle whoſe Diameter is 1, Therefore

The Analogy is,

As 14 is to 11, or as 1 is to $\frac{1}{7}$ and $\frac{1}{2}$ of $\frac{1}{7}$ ſo is the ſquare of the Diameter to the Content. Now $\frac{11}{14}$ wants $\frac{3}{14}$ of the unity, that is $\frac{1}{7}$ and $\frac{1}{2}$ of that, therefore

The Rule is this, viz.

Take $\frac{1}{7}$ of the ſquare of the Diameter; and the $\frac{1}{2}$ of that, and ſubtract their total from the ſquare of the Diameter.

Example.

A Circle whoſe Diameter is 4 foot 5 inches, I demand the ſuperficial Content?

foot.

foot inches.

4 — 05

4 — 05

17 — 08 — 00

1 — 10 — 01

19 — 06 — 01 square of the Diameter.

2 — 09 — 05 — $\frac{2}{7}$ the $\frac{1}{7}$

1 — 04 — 08 — $\frac{4}{7}$ — $\frac{1}{2}$. The half of that.

4 — 02 — 01 — $\frac{6}{7}$ — $\frac{1}{2}$ their total, which subtract- (ed from the squ. of the diam.

Leaves- 15 — 03 — 11 — 0 — $\frac{1}{2}$ for the Area or Content of the Circle.

Or more brief thus, *viz.*

Multiply the Diameter by 3—5—7—8—6—10.

3 — 5 — 7 — 8 — 6 — 10

4 — 5

13 — 10 — 6 — 16 — 3 — 04 — 0

1 — 05 — 4 — 02 — 6 — 10 — 2

15 — 03 — 11 — 00 — 10 — 2 — 2 *Facit.*

Note, *If you take the Diameter by the second line (called the* Diameter square foot) *before directed to be put on the Carpenters Rule, you need do no more but square the Diameter, and you will have the Area of the Circle.*

Prob.

Problem 4.

By the Diameter and Circumference, to find the Area of a Circle.

The Rule.

Multiply the ſemi-diameter by the ſemi-circumference.

Example.

foot inches.
A Circle whoſe diameter is 4 —— 5, and the circumference 13 foot 10 inches $\frac{4}{7}$ what is its Area?

2 —— 2 —— 6 ſemi-diameter.
6 —— 11 —— $\frac{2}{7}$ ſemi-circumference.

13 —— 03 —— 0
2 —— 00 —— 3 —— 6
3 —— 9 —— $\frac{3}{7}$
3 —— 9 —— $\frac{3}{7}$

facit. 15 —— 03 —— 11 —— 0 —— $\frac{6}{7}$

Problem 5.

To find the Diameter by the Content.

The Rule.

Multiply the Area or Content by 14, and divide the product by 11, and the ſquare Root of the quotient will be the Diameter.

Or you many add the $\frac{3}{11}$ of the Area unto the Area (according to the 7th. Rule in Diviſion,) and the ſquare Root of the Total will be the Diameter.

Example.

A Circle whoſe Area or Content is 15—3—11—0—$\frac{6}{7}$, I demand the Diameter.

By the firſt way.

15 — 03 — 11 — 0 — $\frac{6}{7}$
2

30 — 07 — 10 — 1 — $\frac{5}{7}$
7

11)214 — 06 — 11 — 0 — 0

19 — 06 — 01

Facit. — 4 — 05 ſquare Root for the Diameter.

3 — 06 — 01
08 — 05

Here I multiply by the ratio's of 14, *viz.* 2 and 7.

By *the ſecond way.*

15 — 03 — 11 — 0 — $\frac{6}{7}$
1 — 04 — 08 — 7 — $\frac{5}{7}$
1 — 04 — 08 — 7 — $\frac{5}{7}$
1 — 04 — 08 — 7 — $\frac{5}{7}$

19 — 06 — 01 — 0 — 0

Facit. 4 — 05. The ſame again.

3 — 06 — 01
08 — 05

Prob.

Problem 6.

To find the Circumference by the Content.

The Rule.

Firſt find the Diameter by the 5th. Problem and then the Circumference by the firſt.

Example.

I take the ſame Example above, *viz.* a Circle whoſe Area or Content is 15—03—11—0—$\frac{5}{7}$, whoſe Diameter is found to be 4—05, and the circumference I find by the firſt Problem to be 13—10—$\frac{4}{7}$ as in the Operation.

4—05
3—$\frac{1}{7}$

13—03
7—$\frac{4}{7}$

Facit 13—10—$\frac{4}{7}$ Circumference.

Problem 7.

To find the Content by the Circumference.

The Rule.

Firſt find the Diameter by the ſecond Problem, and then the Content by the third.

Example.

A Circle whoſe Circumference is 13—10—$\frac{4}{7}$, I demand the Content?

6 — 11 — $\frac{2}{7}$ Semy Circumference.

7 — $\frac{4}{7}$
7 — $\frac{4}{7}$
7 — $\frac{4}{7}$
7 — $\frac{4}{7}$

2 — 6 — $\frac{2}{7}$

4 — 5 — 0 Diameter.

17 — 8 — 0
1 — 10 — 1

19 — 06 — 1 ſquare of the Diameter.

2 — 09 — 5 — $\frac{2}{7}$
1 — 04 — 8 — $\frac{4}{7}$ — $\frac{1}{2}$

4 — 02 — 01 — $\frac{6}{7}$ — $\frac{1}{2}$

Facit. 15 — 03 — 11 — 0 — $\frac{1}{2}$ ſuperficial Content.

If you multiply the ſemi-circumference by 2-2-5-11-10. it will produce near the ſame.

Problem 8.

To find the ſide of a ſquare that ſhall be inſcribed within a Circle, by the Diameter.

The

The Analogy.

As 5 is to 7 ſo is the ſemi-diameter of the Circle to the ſide of the ſquare that ſhall be inſcribed within the Circle. Or (which is the ſame) as 1 is to 1 $\frac{2}{5}$, ſo is the ſemi-diameter to the ſide of ſuch a ſquare. Therefore multiply the ſemi-diameter by 1 $\frac{2}{5}$.

Example.

foot. inches.

A Circle whoſe Diameter is 4 —— 05. I demand the ſide of a ſquare that may be inſcribed within a Circle of that diameter.

4 —— 05

2 —— 02 —— 06 ſemi-diameter.

5 —— 03 —— $\frac{3}{5}$

5 —— 03 —— $\frac{3}{5}$

Facit.—3 —— 01 —— 01 —— $\frac{1}{5}$ the ſide of the ſquare.

Note, *The third line before directed to be put on the Carpenters Rule*, (called the diameter ſquare inſcribed foot) *gives you the ſide of the ſquare inſcribed only by laying it on the Diameter.*

Problem 9.

To find the ſide of a ſquare equal to a Circle by the Diameter.

The Rule.

Firſt find the Content by the third Problem, and the ſquare root thereof is the ſide of the ſquare equal.

Example.

A Circle whoſe Diameter is 4 foot 05 inches; I demand the ſide of a ſquare equal.

The Area of the Circle is found by the third Problem to be 15 — 03 — 11 and ſome what more. The ſquare root whereof is 3—10—11—9—7.

But more brief, you may multiply the diameter by $10' — 7'' — 7''' — 10''''$, and the product is the ſide of a ſquare equal.

Operation.

	inches	′	″	‴	
	10 —	7 —	7 —	10	
				4 —	5
3 —	06 —	6 —	7 —	4	
	4 —	5 —	2 —	3 —	2
3 —	10 —	11 —	9 —	7 —	2 *Facit*

Note, *The ſecond Line directed to be put on the Carpenters Rule, lain on the Diameter gives the ſide of a ſquare equal to the Circle.*

Problem 10.

To find the ſide of a ſquare equal to a Circle by the Circumference.

The

The Rule.

Multiply the Circumference by 0—3—4—7—5, and the product gives the ſide of a ſquare equal.

Theſe Multiplyers are found according to the queſtions in the end of Diviſion.

Example.

A Circle whoſe circumference is 13—10—06—10, I demand the ſide of the ſquare equal?

```
            13 —— 10 —— 06 —— 10
             · —— 03 —— 04 —— 07 —— 05
         ———————————————————————————————
             3 —— 05 —— 07 —— 08 —— 6
                   4 —— 07 —— 06 —— 3—04—0
                         08 —— 01 —— 1—11—10—0
                               05 —— 9—04—10—2
      ——————————————————————————————————————————
      Facit—3 —— 10 —— 11 —— 09 —— 8—08—08—2
```

(The ſame again being of the ſame Circle.

Note, *The 5th. line on the ſaid Rule taken on the Circumference, gives the ſide of a ſquare equal.*

Problem 11.

To find the Diameter by the ſide of the inſcribed ſquare.

Analogy.

As 7 is to 5, ſo is the ſide of the inſcribed ſquare to the ſemi-diameter. Wherefore multiply the ſide of the ſquare by 5 and divide the product by 7, and double the quote.

Example.

The ſide of an inſcribed ſquare 3 — 01 — 01 — $\frac{1}{3}$, I demand the diameter ?

$$
\begin{array}{r}
3 — 01 — 01 — \frac{1}{3} \\
5 \\
\hline
7)\ 15 — 05 — 06 — 0 \\
\hline
2 — 02 — 06 \\
2 \\
\hline
\end{array}
$$

Facit. — 4 — 05 — 00 Diameter.

Note, *Find the number of the inſcribed ſquare in foot meaſure, on the third Line of the ſaid Rule, and compare that length with your foot meaſure, and you will have the Diameter,* &c.

Problem 12.

To find the Circumference by the ſide of the ſquare inſcribed.

Multiply the inſcribed ſquare by $4 — 5' — 10'' — 5'''$.

Problem 13.

To find the ſide of the inſcribed ſquare by the Circumference.

Analogy.

As 1 is to $\frac{45}{55}$, ſo is $\frac{1}{4}$ of the Circle to the ſide of the inſcribed ſquare. Now $\frac{45}{55}$ wanting $\frac{6}{55}$ of the unity $\frac{5}{55}$ is $\frac{1}{11}$ of

of the unity, and $\frac{1}{33}$ is $\frac{1}{3}$ of that $\frac{1}{11}$, therefore you may take the 11th. of the $\frac{1}{4}$ of the Circumference, and the $\frac{1}{3}$ of that $\frac{1}{11}$, and ſubduct their total from the $\frac{1}{4}$ of the Circumference.

Example.

The Circumference of a Circle is 13—10—06—10, I demand the ſide of the inſcribed ſquare.

13—10—06—10

3—05—07—08—06

03—09—05—01—07
00—09—01—00—03

04—06—06—01—10

Facit—3—01—01—02—04—02 ſide of the inſcribed (ſquare.

Note, *The fourth Line on the ſaid Rule taken on the Circumference, gives the ſide of the ſquare inſcribed.*

Problem 14.

To find the ſide of a ſquare equal to a Circle, by the Area or Content of the Circle.

Rule.

Extract the ſquare root of the Content, and that is the ſide required.

Prob.

Problem 15.

By the ſide of the ſquare to find the Circumference of a Circle that ſhall be equal in Area, to the Area of ſuch a ſquare.

Rule.

Firſt find the Content of the ſquare, by multiplying the ſide into its ſelf, then find the Circumference by the 5 th. and the 6th. Problem.

Example.

The ſide of a ſquare being 3 foot 10 inches, I demand the circumference of a circle that ſhall be equal in Area to the Area of ſuch a ſquare.

Foot inches
3 —— 10
3 —— 10
11 —— 06
3 —— 02 —— 04 the product by 10 inches.
14 —— 08 —— 04 content of the ſquare.
1 —— 04 —— 00 —— 04 —— $\frac{4}{11}$
1 —— 04 —— 00 —— 04 —— $\frac{4}{11}$
1 —— 04 —— 00 —— 04 —— $\frac{4}{11}$
18 —— 68 —— 05 —— 01 —— $\frac{1}{11}$
ſquare root—4 —— 03 —— 10. Diameter.
2 —— 08 —— 05
08 —— 03
0 —— 07 —— 08 —— 01 —— 01
00 —— 08 —— 06 —— 10
: —— 00 —— 06 —— 04 —— 09

4 —— 03

4 —— 03 —— 10 Diameter.
3 —— $\frac{1}{7}$

12 —— 11 —— 06
7 —— 04 —— $\frac{6}{7}$

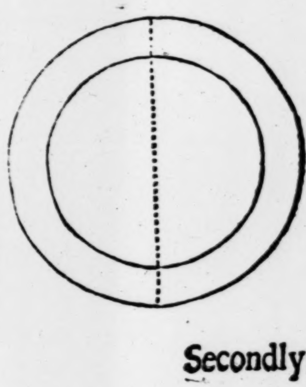
13 —— 06 —— 10 —— $\frac{6}{7}$ circumference.

Note, *If you find the number of the side of your square in foot measure, on the 5th. Line of the said Rule, and compare that length with your foot measure, you will have the Circumference,* &c.

Note again, *If the Area of a Circle be given to find the Circumference, Diameter, square equal, or square inscribed; find the square root of the fixed Area, and of the given Area. Then by the Rule of Three Direct, say as one square root is to the other, so is the fixed Diameter circumference square equal, or inscribed to the enquired Diameter circumference square equal, or inscribed. But having given sufficient instructions for this in the Rule of Three Direct, I refer you thereunto.*

Fourthly, *Of Segments of Circles.*

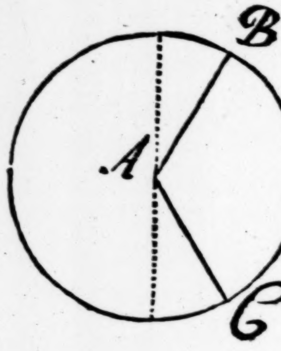
First, WHen the Segment runs Circular, and Parallel with the center.



Find the Area of the outer circle, and then find the Area of the inner by the third problem, and subtract the Area of the inner from the Area of the outer. This is so plain as it needs no Example.

Secondly,

Secondly, When the segment runs from the circumference to the center, having two semi-diameters, and one Arch line, As A B C in the Figure.



Multiply $\frac{1}{2}$ the Arch line by the semi-diameter.

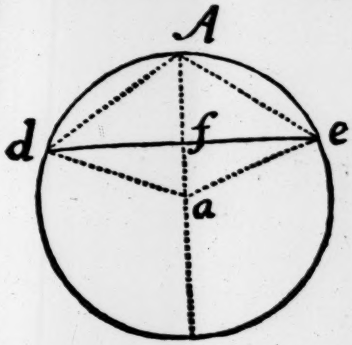
Example.

foot inches.

The whole diameter being 2 --- 6 the half is 1 —— 03. which multiplyed by 1—3—9 (the half Arch of $\frac{1}{3}$ part of the Circle) hath —1—7—8—3 for the Area of $\frac{1}{3}$ part of the Circle.

1	03	09		the half Arch.
1	03	00		the semi-diameter.
1	03	09		
0	03	11	3	
1	07	08	3	*Facit.*

Thirdly,



Thirdly, To find the Area of the ſegment of a circle that ſhall croſs the diameter at right Angles which may be called a cantell, as the ſegment A *d e* in the figure.

Firſt, Find the Area of the whole ſegment running to the center as A *d e a* by the laſt problem, in which you have an Iſoſceles Triangle *d e a*, and the cantell A *d e*, therefore find the Area or content of the Triangle, and ſubtract it from the Area of the whole ſegment A *d e a* before found, and the remain is the Area of the cantel required. To find the perpendicular of the Triangle; ſubtract the arrow or verſed ſine from the diameter, and the remain is the perpendicular.

Example.

Let 19 — 06 — 00 — 00 be the Diameter.
22 — 09 — 11 — 06 — 02 Arch line.
18 — 00 — 00 — 00 — 00 Chord of the ſegment.
6 — 00 — 00 — 00 — 00 Arrow or verſed ſine.
3 — 09 — 00 — 07 — 05 Perpendicular.
9 — 00 — 00 — 00 — 00 half the Chord.

Now

11—04—11—09—01 half the Arch line.
multiplied by 9—09—00—00—00 the ſemi-diameter.

102—08—09—09—09
5—08—05—10—06—06
2—10—02—11—03—03

produceth - 111—03—06—07—06—09 for the whole ſegments area.
Then — 33—09—05—06—09 the Area of the Triangle Subtracted

Leaves 77—06—01—00—09—09 for the Area of (the ſegment required.

Here

Here Note three Things.

Firſt, TO find the diameter by the Chord of the ſegment do thus, firſt ſquare half the Chord of the ſegment *d e*, and divide the product by the ſegments Altitude, or ſine A *f*, then the quote, and the ſine added is the whole diameter of the anſwerable circle.

Thus *d f* being 9, ſquared makes 81, then 81 divided by 6 the ſegments Altitude *A f* quotes 13 — 06, to which adding 6 makes 19 — 6 the Diameter.

Secondly, To find the Chord *d a* of half the Arch of the ſegment.

Add the ſquare of the ſemi-chord of the ſegment *d f* with the ſquare of the ſine *A f*, and the ſquare root of the total gives the chord of half the ſegments Arch.

Thus the ſquare of 9 is 81, aud the ſquare of 6 is 36, which added makes 117, the ſquare root whereof is 10-9-8.

Thirdly, To find the length of the ſegments Arch very near.

Find the difference between 18 (*viz*. *d e*) the ſegments Chord, and the ſumme of *A d*, and *A e* the two chords of the ſegments Arch, *viz*. 10—9—8—7, and —10—9—8-7, which makes 21—7—5—2, and the difference will be 3—7—5—2, one third whereof is 1—2—5—8—8. Then the ſumme of the two chords, *viz*. 21—7—5—2, and the $\frac{1}{3}$ part of the difference 1—2—5—8—8 being added makes 22—9—10—10—8 the length of the Arch, the half whereof multiplyed by half the diameter, gives the Area of the whole ſegment, and the Area of the Triangle deducted gives the Area of the ſegment required as above.

Fourthly, To divide the Circumference of any Circle into any parts required.

Di-

Divide 360 by the number of parts required, the quotient giveth the Chord that will divide the circumference into the parts required.

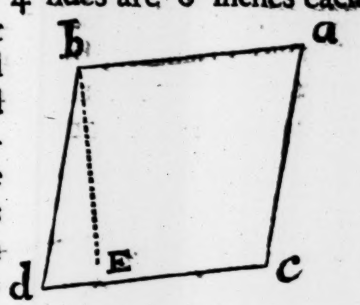
Fifthly, To meaſure a Rhombus or Rhomboides.

A Rhombus is a Geometrical ſquare bent awry, as a long quarry of Glaſs, and a Rhomboides is a long ſquare or parallelogram out of ſquare at the ends, as the figures ſhew, and are thus meaſured, *viz.*

Multiply one ſide in the Rhombus or Rhomboides by the perpendicular or neareſt diſtance between the ſides, and that product ſhall be the content of the Rhombus or Rhomboides.

Example.

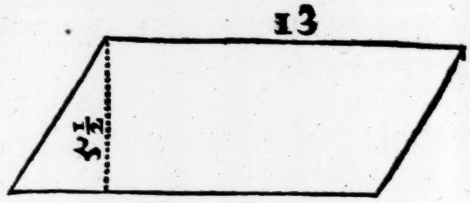
There is a Rhombus, whoſe 4 ſides are 6 inches each, but the Angles or corners at *a b c d* are not equal, but *a* and *d* are acute or ſharp Angles, and *b* and *c* are obtuſe or blunt Angles, but the neareſt diſtance from *a b* to *c d* is *b E* 5 $\frac{1}{2}$ inches though *b* and *d* be 6 inches, then I ſay, the product of 5 $\frac{1}{2}$ multiplyed by 6 is the ſuperficial content of the Rhombus, *viz.* 33 which in the ſquare of the ſame meaſure was 36.



$$\begin{array}{r}5 \text{ — } \frac{1}{2}\\ 6\\ \hline 33 \text{ — } 0\\ \hline\end{array}$$

Alſo

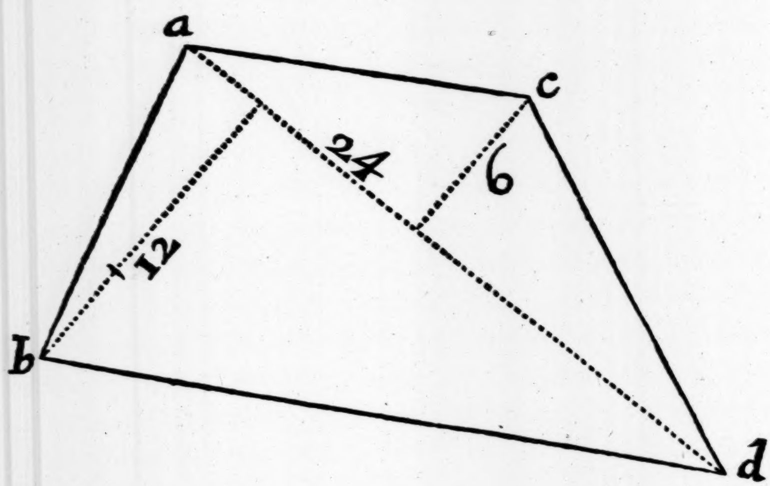
Alſo in the Rhomboides, 13 the length of one ſide multiplyed by 5 $\frac{1}{2}$ gives 71 $\frac{1}{2}$ for the ſuperficial content.



13	—
5	$\frac{1}{2}$
65	—
6	$\frac{1}{2}$
71	$\frac{1}{2}$

Seventhly, To meaſure a Trapezia.

A Trapezia is a Figure of any 4 unequal ſides as the Figure ſheweth, and the meaſuring of it is beſt done by bringing it into two Triangles, by drawing a Line from corner to corner as in the figure, then the meaſure of thoſe two Triangles is the true Area or content of ſuch a Figure.



Exam-

Example.

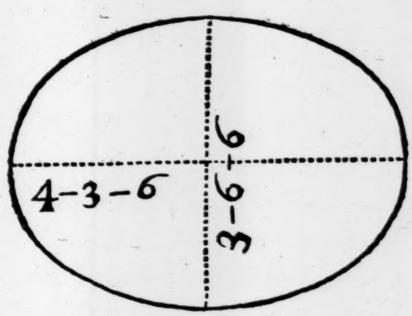
Let *a b c d* be a Trapezia, and the line *a d* drawn from *a* to *d* be 24 inches, and the neareſt diſtance from that line to *c* one corner 6 inches, and from *b* the other corner to that line continued 12 inches, then 12 the half of the Diagonal line, and 18 the ſumme of the two perpendiculars multiplyed, the product 216 is the ſuperficial content.

Or without finding the perpendiculars, thus,

1. From the ſemi-perimeter of a Triangle ſubduct each ſide ſeverally.
2. Multiply the ſemi-perimeter, and the 3 differences aforeſaid by each other according to continual multiplication.
3. The ſquare root of that product is the Triangles Area.
4. The ſumme of the Triangles Areas ſo found, is the Area of the Trapezium.

Eighthly, To meaſure an Ellipſis or Oval.

Multiply the greater Diameter by the leſs, and take $\frac{1}{7}$ of the product, and the $\frac{1}{2}$ of that, and ſubtract their total from the product.



Example.

There is an Ovall or Ellipſis, whoſe greater diameter is 4—3—6, the leſſer diameter is 3—6—6. I demand the ſuperficial content.

4 — 3 — 6
3 — 6 — 6

12 — 10 — 6
2 — 01 — 9
02 — 1 — 9

15 — 02 — 4 — 9 product.

2 — 02 — 0 — 8 — $\frac{1}{7}$
1 — 01 — 0 — 4 — 0 — $\frac{1}{2}$

3 — 3 — 1 — 0 — $\frac{1}{7}$ — $\frac{1}{2}$

11 — 11 — 3 — 8 — $\frac{5}{7}$ — $\frac{1}{2}$ *facit.*

Note, *When you have multiplyed the greater Diameter by the lefs, the fquare root of the product will be the diameter of a Circle that fhall be equal in Area with the Ellipfis, and then having the diameter of a Circle equal, the various cafes are wrought as formerly in the Circle into which figure the Ellipfis is thus reduced.*

Therefore

Only for the finding of the Area of the fegment of an Ellipfis, take this following Analogy, *viz.*

Find the correfpondent Area of the fegment of a Circle, anfwerable to the greater diameter of the Ellipfis, and then

As the diameter of the Circle is,
To the leffer diameter of the Ellipfis,
So is the Area of the fegment of the circle
To the correfpondent Area of the Ellipfis fegment fought.

Here

Here follows ſome Queſtions concerning Circles, to put the foregoing Problems in Practice.

1. *Queſtion.*

THree Men bought a circular Cheeſe 14 inches diameter, which coſt them 7 s. 6 d. Whereof

	s.	d.
A payes	1	04
B payes	2	10
C payes	3	04

And the Cheeſe being to be divided proportionably between them, they agreed, it ſhould be divided from the circumference to the center. Now the Queſtion is how many inches of the circumference each man muſt have, and how many ſquare inches ſuperficial his part will be?

Reſolution.

Firſt, Find the circumference by the firſt problem of Circles.

Secondly, Find the Area or content of the whole Cheeſe in inches by the third problem.

Thirdly, Find each mans part of the Area by the Rule of proportion, thus, *viz.* As the whole price is to the whole Area, ſo is each mans particular price, to his particular Area. Then

Fourthly, Find each mans part of the circumference thus, *viz.* As the whole Area is to the whole circumference, ſo is each mans particular Area, to his particular part of the circumference.

Operation.

14
3 — $\frac{1}{7}$

42
2

inches 44 circumference.

14
14

56
14

196

28
14

42

inch. 154 Area.

s. d. inch. s. d.
7—6—154—1—4
51—4

7—6) 205—4 (27 $\frac{2—10}{7—6}$ *A.*

20-00
15-00

55-04
52-06

·2-10

s. d. inch. s. d.
7—6—154—2—10
2-10

308
77
51-04

7—6) 436-04 (58 $\frac{1—4}{7—6}$ *B.*

43-00
37-06

·5-06

61-04
60-00

·1-04

s. d. inches
7—6 — 154 — 3 — 4
3 — 4

462
51 — 4

7—6) — 513 — 4 (68 $\frac{3—4}{7—6}$ C

51 — 00
45 — 00

63 — 04
60 — 00

3 — 04

inches.
27 — 2 — 10 For A.
58 — 1 — 04 For B.
68 — 3 — 04 For C.

154 — 7 — 06 Proof.

Note, *the* 7 — 6 *in the Duodecimal places making the Denominators, they muſt be reckoned* 1 *Integer to be carried to the Inches.*

Area Circumf. Area.

If 154 give 44 —— what 27 $\frac{2—10}{7—06}$

7—6 7

1078 189

77 13 — 06

1155 2 — 10

205 — 04

44

820

8200

14 — 8 inches

1155.) 9034 — 8 (7 $\frac{949—8}{1155—0}$ A.

8085 — 8

·949 — 0

1155 — 44 — 58 $\frac{1—4}{7—6}$

7

406

29

1 — 4

436 — 4 (

44

1744

17440

14 — 8

inches

1155.) 19198 — 8 (16 $\frac{718—8}{1155—0}$ B.

1919

1155

·7648 — 8

6930 — 0

·718 — 8

1155—44—68 $\frac{3—4}{7—6}$
7

476
34
3—4

513—4
44

2052
20520
14—8
inches
1155.) 22586—8 (19 $\frac{641—8}{1155—0}$ C.

2258
1155

11036—8
10395—0

..641—8 So that

		inch. Area.		inch. Circumf.
A	muſt have	27 $\frac{2—10}{7—6}$	And	7 $\frac{949—8}{1155—0}$
B		58 $\frac{1—4}{7—6}$		16 $\frac{718—8}{1155—0}$
C		68 $\frac{3—4}{7—6}$		19 $\frac{641—8}{1155—0}$

7—949—8——A.
16—718—8——B.
19—641—8——C.

44—2310—0——Proof.

2. *Queſtion.*

A B and C bought a Grinding ſtone of 21 inches Diameter, each paying $\frac{1}{3}$ part. *Queſtion*, what part of the Diameter muſt each grind, A grinding firſt, B next, and C laſt?

Reſolution.

Firſt, Find the Area in ſquare inches by the third Problem.

Secondly, Find each mans $\frac{1}{3}$ part of the Area.

Thirdly, Subtract $\frac{1}{3}$ of the Area from the whole, and then find the Diameter of the remaining $\frac{2}{3}$ Area, by the 5th. Problem, which call the ſecond Diameter.

Fourthly, Subtract that Diameter from the firſt Diameter, and the remain is the proportionable part of the Diameter for A.

Fifthly, Find the Diameter of the other $\frac{1}{3}$ Area, by the 5th. Problem, and ſubtract that Diameter from the ſecond Diameter, and the remain is B his proportionable part of the Diameter and that which you ſubtracted will be C's.

inches.

Operation.

inches.
21 Diameter.
21
———
21
42
———
441
———
63
31 — 6
———
94 — 6
———
346 — 6 whole Area.
———
115 — 6 the $\frac{1}{3}$ part.
———
231 — 0 the $\frac{2}{3}$ parts.
7
———
1617
2
———
11) 3234
———
294
———
ſq. root 17 — 01 — 09 ſecond Diameter of the $\frac{2}{3}$ Area.
———
194
27
———
5 — 0 — 0
2 — 10 — 1
———
2 — 01 — 11 — 0 — 0
2 — 10 — 2 — 9
———
[illegible] — [illegible] — 2 — 11 — 3 $\frac{2}{3}$ Area

$\frac{1}{3}$ Area 115 —— 06
7

808 —— 06
2

11) 1617 —— 00

141

Diameter of the $\frac{1}{3}$ Area 12 —— 01 —— 05 square root.

47
22

3 —— 00 —— 00
2 —— 00 —— 01

—— 11 —— 11 —— 00 —— 00
2 —— 00 —— 02 —— 05

1 —— 09 —— 11 —— 11

inches.

Diameter 21 —— 00 —— 00 of the grinding stone.
Diameter 17 —— 01 —— 09 of the $\frac{2}{3}$ Area subtracted.

Leaves 3 —— 10 —— 03 of the Diameter for A.

2 Diam. —— 17 —— 01 —— 09 of the $\frac{2}{3}$ Area, from which
Diameter 12 —— 01 —— 05 of the $\frac{1}{3}$ Area subtracted.

Leaves 5 —— 00 —— 04 of the Diameter for B.

And 12 —— 01 —— 05 of the Diam. is for C. So that
A is to grind 3 inch. 10 —— 03
B is to grind 5 —— 00 —— 04
C is to grind 12 —— 01 —— 05

21 —— 00 —— 00 *Proof.*

3. *Questi-*

3. *Queſtion.*

6 Men buy a Grinding Stone of 36 inches Diameter, paying as followeth, *viz.*

	s.
The firſt	11
ſecond	09
third	07
fourth	05
fifth	03
ſixth	01
In all	36 s.

Queſtion, how many inches of the Diameter muſt each man grind for his proportion, grinding one after another as they are placed?

Reſolution.

Firſt, Find the whole Area by the Third Problem.

Secondly, Find each Mans part of the Area by the Rule of Proportion.

Thirdly, Subtract the firſt Mans Area from the whole Area, and call the remain, the firſt remaining Area, and find the Diameter of a Circle of that remaining Area by the fifth Problem; which call the ſecond Diameter, and ſubtract that Diameter ſo found from the whole Diameter, and that which remains are the inches of the Diameter that the firſt Man muſt grind.

Fourthly, Subtract the ſecond Mans Area from the firſt remaining Area, and call the remain the ſecond remaining Area, and find the Diameter of a Circle of that remaining Area by the fifth Problem, which call the third Diameter, and ſubtract it from the ſecond Diameter, and the remain are the inches that the ſecond Man muſt grind.

Fifthly, Do the ſame until you come to the laſt, and you ſhall find that each Man muſt grind 6 inches of the Diameter, grinding one after the other, as they are placed above.

Operation.

Operation. 1018 — 3 — 5 — $\frac{1}{7}$ whole Area.

If 36 (s.) must have 1018-3-5-$\frac{1}{7}$ (*Area.*) what
$\left\{\begin{array}{c} 11 \\ 9 \\ 7 \\ 5 \\ 3 \\ 1 \end{array}\right\}$ *Facit*
$\left\{\begin{array}{l} 311-\ 1-\ 8-\frac{4}{7} \\ 254-\ 6-10-\frac{2}{7} \\ 198-\ 0-\ 0-0 \\ 141-\ 5-\ 1-\frac{5}{7} \\ 084-10-\ 3-\frac{3}{7} \\ 028-\ 3-\ 5-\frac{1}{7} \end{array}\right.$

Proof — 1018- 3- 5-$\frac{1}{7}$

1018 — 3 — 5 — $\frac{1}{7}$
311 — 1 — 8 — $\frac{4}{7}$ first Mans Area subtracted

707 — 1 — 8 — $\frac{4}{7}$ first remaining Area.
14

11) 9900 — 0 — 0 — 0 Product.

900 — 0 — 0 — 0 Quotient.

Sq.Root 30 Second Diameter 36
30

6 for the first Man.

707 — 1 — 8 — $\frac{4}{7}$
254 — 6 — 10 — $\frac{2}{7}$ sec. Mans Area subtracted.

452 — 6 — 10 — $\frac{2}{7}$ second remaining Area.
14

11) 6336 — 0 — 0 — 0 Product.

576 — 0 — 0 — 0 Quotient.

Sq.root 24 Third Diameter 30
24

6 for the second Man.

452

$452 — 6 — 10 — \frac{2}{7}$
$198 — 0 — 0 — 0$ third Mans Area subtracted.

$254 — 6 — 10 — \frac{2}{7}$ third remaining Area.
14

11) $3564 — 0 — 0 — 0$ Product.

$324 — 0 — 0 — 0$ Quotient.

Sq.root 18 Fourth Diameter. 24
18

6 for the third.

$254 — 6 — 10 — \frac{2}{7}$
$141 — 5 — 1 — \frac{5}{7}$ 4th. Mans Area subtracted.

$113 — 1 — 8 — \frac{4}{7}$ fourth remaining Area.
14

11) $1584 — 0 — 0 — 0$ Product.

$144 — 0 — 0 — 0$ Quotient.

Sq.root 12 Fifth Diameter. 18
24

6 for the fourth.

113 — 1 — 8 — $\frac{4}{7}$
84 — 10 — 3 — $\frac{3}{7}$ fifth Mans Area ſubtracted.

28 — 3 — 5 — $\frac{1}{7}$ fifth remaining Area.
14

11) 396 — 0 — 0 — 0 Product.

36 — 0 — 0 — 0 Quotient.

Sq.Root *6* Sixth Diameter. 12
6

6 for the fifth.

And *6* more remains for the ſixth Man.

Ninthly, To Meaſure a many ſided irregular Figure.

Theſe are to be reduced into Triangles or Trapeziaes by drawing of Diagonal lines from one corner to another, and then meaſured and caſt up ſeverally by the Rules aforegoing for a Triangle and Trapezia.

Tenthly, To Meaſure a Regular Poligon.

A Regular Poligon is when the ſides are more than 4, and yet all equal, as of 5, 6, 7, 8, 9, or 10, equal ſides, and the way to Meaſure it is thus, *viz.*

Multiply $\frac{1}{2}$ the ſum of all the ſides by the length of the line drawn from the Center to the middle of any one ſide, and the product will be the ſuperficial Content.

Eleventhly,

Eleventhly, To make a ſquare, two, three, four, or any number of times bigger than another ſquare.

Square the ſide thereof to find the content, then double, treble or quadruple the content as you have occaſion, and the ſquare root thereof extracted ſhews the ſide of the ſquare deſired.

So having the Diameter, Semy-Diameter, or the circumference of a Circle to make another 2, 3, or 4 times bigger than another Circle ; ſquare it, then double, treble, or quadruple the number, and extract the ſquare root thereof ; ſo you ſhall have the Diameter, Semy-Diameter, or circumference of a Circle that ſhall be 2, 3, or 4 times bigger than the other.

Twelfthly, The ſide of a ſqnare being given to find the Diagonal.

Double the ſquare of the ſide, and the ſquare root thereof is the Diagonal.

II. *Of Superficies of Solid Bodies.*

THus far for the finding the ſuperficial content of plain ſuperficies : Now for the finding the ſuperficial content of the ſuperficies of ſolid Bodies.

Firſt, *Of Round Pillars.*

The Rule.

Multiply the length by the circumference, and the product will be your deſire.

Example.

Example.

A Painter hath painted a pillar that is 5 foot 10 inches in circumference, and 12 foot 8 inches long. Queſtion, how many yards ſquare is the painting?

Note, I divide the feet by 9, becauſe 9 ſquare feet are in a ſquare yard. Or you may work by the yard, as before directed.

	foot	inches.		
	12	08		
	5	10		
	63	04		
	6	04		
	4	02	08	
9)	73	10	08	
yrds.	8	02	06	$\frac{2}{9}$ *Facit.*

Secondly, *Of Square Pillars.*

The Rule.

Add the 4 breadths together, and multiply the Total by the length.

Thirdly, *Of Regular Poligons.*

Add all the breadths together, (be they of as many ſides as they will,) and multiply the total by the length, and the product gives your deſire.

Fourthly, *Of Cones.*

Multiply $\frac{1}{2}$ the circumference at the Baſe by the length, or $\frac{1}{2}$ the length by the circumference.

Fifthly,

Fifthly, *Of Pyramids.*

Add all the breadths at the baſe together, and multiply $\frac{1}{2}$ the total by the length, or $\frac{1}{2}$ the length by the total.

Sixthly, *Of Globes or Spheres.*

Multiply the Spheres Diameter by the Circumference of the ſame, and the product is your deſire, Or,

Multiply the Area of the Circle by 4, and it will produce the Content.

Seventhly, *To find the Superficial Content of the Segment of a Globe.*

Multiply the Circumference of the Sphere by the ſegments altitude, and the product is the ſuperficial Content.

Secondly, *Of Solids.*

HAving fully inſtructed you in meaſuring of Superficies, I come now to treat of meaſuring of Solids.

Firſt, *Of Timber.*

And here Note that Timber is meaſured by the cubick or ſolid foot, that is to ſay a foot every way, or if you will a foot long, a foot broad, and a foot deep. And the cubick foot is divided in the breadth into 12 equal parts, which I call primes or inches of the cubick foot; each ſuch inch containing a ſquare foot (12 of which makes the cubick or ſolid foot.) Then it is divided in its length into 12 parts

more, cutting each of the foreſaid primes or inches into 12 parts alſo, which I call ſeconds, or parts of the inch of the cubick foot, (12 of which makes one of the ſaid primes or inches) and each of thoſe parts is 12 inches long, and one inch ſquare: Then it is divided again in the depth into 12 other parts, cutting each of the foreſaid parts into 12 parts alſo, which I call thirds, or parts of a part (12 of which makes one of the former parts,) and each of thoſe thirds or parts of a part is a cubical inch, *viz.* one inch ſquare, or one inch every way. And ſo thoſe parts may be divided again, *ad infinitum.*

A few Examples will be ſufficient for your inſtruction. And here,

First. *Of Square Timber.*

1. *Example.*

A piece of Timber 14 foot long, and 8 inches ſquare, how many foot ſolid doth it contain?

The Rule.

Multiply the length by the breadth, and the product by the depth.

Operation.

```
      foot.
       14 —— 00
     ——————————
        4 —— 08
        4 —— 08
     ——————————
        9 —— 04
     ———————————————
        3 —— 01 —— 4
        3 —— 01 —— 4
     ———————————————
Facit   6 —— 02 —— 8
     ———————————————
```

8 inches being $\frac{2}{3}$ of 12, I take the $\frac{2}{3}$ of the length and add them together for the length by the breadth, then I do the like by the product for the depth.

Or

Or you may find the Area of the ſquare plane at the end, and multiply the length thereby, as in the Operation.

Operation.

Inches.
8 ——— by 8 inches.

2 ——— 8
2 ——— 8

5 ——— 4 Area.

14 ——— 00 by the Area.

4 ——— 08
1 ——— 02
04 ——— 8

The ſame again 6 ——— 02 ——— 8

2. *Example.*

A piece of Timber 15 foot 8 inches and half long, and 10 inches and half ſquare, how many ſolid feet doth it contain?

Operation.

The firſt way.

```
                          foot inches parts.
                          15 —— 08 —— 06
                          ————————————————
                           7 —— 10 —— 03
                           5 —— 02 —— 10
                                07 —— 10 — 03
                          ————————————————————————
product of the length     13 —— 08 —— 11 — 03
by the breadth            ————————————————————————
                           6 —— 10 —— 05 — 07 — 6
                           4 —— 06 —— 11 — 09 — 0
                                06 —— 10 — 05 — 7 — 06
                          ——————————————————————————————
                Facit. 12 —— 00 —— 03 — 10 — 1 — 06
                          ——————————————————————————————
```

The ſecond way.

```
                    inches parts   inches parts.
                      10 —— 06 by 10 —— 06.
                    ——————————————
                      5 —— 03
                      3 —— 06
                           05 —— 3
                    ——————————————————
Area of the plane     9 —— 02 —— 3
                    ——————————————————
                     15 —— 08 —— 6 The length.
                    ——————————————————
                      7 —— 10 —— 3
                      3 —— 11 —— 1 —— 06
                           02 —— 7 —— 05 — 00
                                 3 —— 11 — 01 — 06
                    ——————————————————————————————
The ſame again.      12 —— 00 —— 3 — 10 — 01 — 06
                    ——————————————————————————————
```

In

In multiplying by the parts, I work according to the third Rule in Multiplication.

Secondly, *Of Timber that is not ſquare, but thicker than broad, called Parallelopipedons.*

1. *Example.*

A piece of Timber 14 foot and half long, 8 inches broad and 9 inches thick, how many ſolid feet doth it contain?

Operation.

The firſt way.

foot	inches.
14	06
4	10
4	10
9	08
4	10
2	05
Facit. 7	03

The ſecond way.

inches.

8 — by 9 inches.

4	foot.	
2	14 —	06 length.
Area of the plane 6	7 —	03 the ſame (again.

But here Note, what advantages may be taken by a diſcerning eye, for if you obſerve what aliquot part of 12 the parts you are to multiply by, are leſs than 12, you may multiply by ſuch aliquot part, and ſubtract the product from the multiplicand, and the remain will be the ſame as if you had multiplyed by the right multiplicator, and for an inſtance I will take the ſame Example, *viz.* A piece 14 foot and half long, 8 inches broad, and 9 inches thick. Now 8 inches is 4 inches leſs then 12, and 9 inches is 3 inches leſs than 12, therefore I take the third for the former, and the fourth for the latter, as in the example.

	foot.	inches.	
	14 —	06. long	
$\frac{1}{3}$ —	4 —	10	ſubtracted.
	9 —	08	remain.
$\frac{1}{4}$ —	2 —	05	ſubtracted.
	7 —	03	remain.

The ſame with the former.

2. *Example*

A piece of Timber 15 foot $\frac{3}{4}$ long, 14 inches broad, and 10 inches and half thick, how many ſolid feet contains it?

	foot.	inches.			
	15 —	9 long			
	2 —	7 —	6		
	18 —	4 —	6		
$\frac{1}{6}$ —	3 —	0 —	9		ſubtracted.
	15 —	3 —	9		remain.
	· —	9 —	2 —	3	
Facit.	16 —	0 —	11 —	3	

Note

Note here, *Having given you 3 ſeveral wayes to work by, which are moſt facile and plain, I ſhall henceforward make uſe of that onely which I ſhall find moſt advantageous. As here inſtead of multiplying by 14 inches, 12 inches making the foot I reckoned 15 foot 9 inches for 1 foot, and then took the* $\frac{1}{6}$ *for 2 inches. And for 10 inches I took* $\frac{1}{6}$ *and ſubtracted it.*

3. *Example.*

A piece of Timber 16 foot 10 inches long, 14 inches $\frac{3}{4}$ broad, and 8 inches $\frac{3}{4}$ thick, how many ſolid feet may it contain.

Foot inches
16 — 10 — long
2 — 09 — 8
08 — 5
4 — 2 — 6

20 — 08 — 3 — 6

6 — 10 — 9 — 2
6 — 10 — 9 — 2
10 — 4 — 1 — 9
5 — 2 — 0 — 10 — 6

Facit. 15 — 01 — 0 — 6 — 07 — 6

Here I have wrought by the plaineſt way, *viz.* the length I have multiplyed by the breadth, and the product by the depth or thickneſs.

 Thirdly,

Thirdly, *Of Timber that may have the ſame breadth, but not the ſame thickneſs throughout, called Fruſtums of Priſms.*

Example.

A piece of Timber 10 foot long, 10 inches and half broad throughout, 9 inches and half thick at one end, and 7 inches and half thick at the other end, how many ſolid feet contains it?

The Rule.

Firſt multiply the length by the breadth as before, then take an Arithmetical mean between the two thickneſſes, *viz.* add them together, and take half of the total for the Arithmetical mean, and work thereby, as if it were of that thickneſs throughout.

inch. parts.
9 — 6
7 — 6

Operation. 17 — 0

8 — 6 Arithmet. mean

foot.
10 — 00 long

5 — 00
3 — 04
05

8 — 09 product of the length by the breadth.

2 — 11
2 — 11
4 — 4 — 6 Multiplyed by the Arithmetical mean.

Facit — 6 — 2 — 4 — 6

Fourth-

Fourthly, *Of Timber that may have thickneſs at one end, or at one ſide, but none at the other end or ſide, but terminates in a line, like the ridge of a houſe.*

1. Example.

A Priſme. A piece of Timber 16 foot long, 10 inches $\frac{1}{4}$ broad, and 11 inches thick at one end, and of no thickneſs at the other end, how many ſolid feet may it contain?

The Rule.

Multiply the length by the breadth as before, and the product by $\frac{1}{2}$ the thickneſs at the end.

Operation.

```
foot                          inch.
 16 — 00 — long.               11 the ½ whereof
 ---------------              ---------------
  8 — 00                      is—5 — 6
  5 — 04
      04
 ---------------
 13 — 08
 ---------------
  4 — 06 — 8
  1 — 01 — 8
      06 — 10
 ---------------
Facit ·6 — 03 — 02
 ---------------
```

2. Exam-

2. Example.

A Prisme.

And a part or segment of either of these is called the frustrum of a Prisme, being cut with a plane Parallel to the Base.

fo. inch.

A piece of Timber 15—8 long, 9 inches $\frac{1}{2}$ broad, and 10 inches 8 parts thick on one side, and of no thickness on the other, how many solid feet contains it?

The Rule.

Multiply the length by the breadth, and the product by $\frac{1}{2}$ the thickness as before.

Operation.

foot	inches					inch.	parts	
15 —	08 long.					10 —	8 the $\frac{1}{2}$ whereof	
7 —	10					is—5 —	4	
3 —	11							
	7 —	10						
12 —	4 —	10						
4 —	01 —	07 —	04					
1 —	00 —	04 —	10					
	4 —	01 —	07 —	04				
5 —	06 —	01 —	09 —	04				

Fifthly,

Fifthly, *Of Timber that may be broader at one end than at the other, and of the ſame thickneſs or depth at both ends.*

Example.

A piece of Timber 8 foot long, 10 inches broad at one end, and 6 inches broad at the other end, and 8 inches thick throughout, how many ſolid feet may it contain?

This may be termed the fruſtrum of a Priſme.

The Rule.

Firſt, Find the Area of the ſuperficies, by multiplying $\frac{1}{2}$ the Diagonal by the total of the two perpendiculars.

Secondly, Multiply the whole Area ſo found by the depth, and it gives the ſolid Content in feet.

But you may perform it better thus; *viz.* Add the two breadths together, and by $\frac{1}{2}$ the total multiply the length, and that product by the depth, and according to this Rule I work the Queſtion.

inches.		foot. inches.	
10 } 6 }	the 2 breadths.	8 — 00	long.
		5 — 04	Product of the breadth.
16	Total.	0 — 8	
8	the $\frac{1}{2}$.	*Facit* 3 — 06 — 8	Product of 8 inches the depth.

Note, In ſome places they ſell Solid or Groſs Timber by the Foot ſuperficial, that is to ſay, they find how many foot of Board meaſure will be contained in the ſolid piece if it were ſawn into inch boards.

Now

Now to work this, do thus, viz.

First, Find the superficial Content of the breadth in feet and inches, as you have been taught.

Secondly, Multiply that by the inches of the depth.

Example.

A piece of Timber 13 foot long, 8 inches broad, and 10 inches thick; How many foot of Board measure doth it contain?

Operation.

Foot	Inches.	
13 ——	00	long.
4 ——	04	
4 ——	04	
8 ——	08	superficial content of the breadth.
	10	
Facit 86 ——	08	whole content in Board Measure.

Note, The $\frac{1}{12}$ of the Board measure, will be the Solid or Cubick feet contained therein, as the $\frac{1}{12}$ of 86 foot, 8 inches of Board measure makes 7 foot, 2 inches, 8 parts solid.

Now for a Conclusion of four sided Timber, I shall give you but this Problem, *viz.*

Having the sides of a piece of Timber squared, to find what length on the same piece shall make a foot solid.

The

The Rule.

Firſt ſquare it, *viz.* multiply one ſide by the other, and if the ſquare amounts to 1 foot or more, the Analogy will be

As the ſquare of one foot (1)
To 12 Inches in length,

So will the ſquare of the piece be to the inches of the length that ſhall make a foot ſolid, in a Reverſe proportion.

But if the ſquare amounts not to one foot, then the Analogy will be,

As the ſquare of one inch (1)
To 144 foot in length,

So will the ſquare of the piece be to the feet of the length that ſhall make a foot ſolid, in a Reverſe proportion alſo.

Example of the firſt.

A piece of Timber of a certain length, whereof one ſide is 1 foot 2 inches, and the other 1 foot 1 inch, what length on the ſame piece will make a foot ſolid?

```
        1 — 2
            1 — 2                   inch.  '    ''
Square  1 — 3 — 2 ) 12 — 0 — 0 ( 9 — 5 — 11 facit.
                    · — 7 — 6 — 0
                          1 — 2 — 2 — 0
                          · — · — 3 — 2
```

To

To prove it, I say,

A piece 14 inches broad, and 13 inches thick, and

inch. ′ ″
9 — 5 — 11 long, what is its solidity?

```
inch.  ′    ″
 9 —  5 — 11
 1 —  6 — 11 — 10
-------------------------
11 —  0 — 10 — 10
     11 —  0 — 10 — 10
-------------------------
11 — 11 — 11 —  8 — 10
                3 —  2 Remain.
-------------------------
Foot. 1 — . — . — . — . — . Facit.
```

Example of the second.

Note, *I make Inches the Integers here.*

A piece of small Timber of a certain length, one side whereof is 1 inch and half, and the other 1 inch and $\frac{1}{4}$, how many foot in length on the same piece will make a foot solid.

```
inches. ′   ″
   1 — 6 — 0
       4 — 6
---------------
                          foot.   ′    ″
square 1 — 10 — 6) 144 (76 — 9 — 7 Facit.
                   14 —  0 —  0
                   ------------
                    · — 10 —  6
                   ------------
                   12 —  9 —  0
                   ----------------
                    1 —  6 —  0 — 0
                   --------------------
                    · —  1 —  1 — 6 — 0
                   ------------------------
                         · —  · — 4 — 6
                         ------------------
```

Note,

Note, *What ever the form be (the Dimenſions being the ſame throughout) find the Area of the plane, and divide as before.*

To prove it I ſay,

A piece 76 foot, 9 primes, 7 ſeconds long, and 1 inch and half broad, and 1 inch $\frac{1}{4}$ thick what is its ſolidity.

```
76 — 9 — 7
─────────────
6 — 4 — 9 — 7
3 — 2 — 4 — 9 — 6
─────────────────
9 — 7 — 2 — 4 — 6
─────────────────
0 — 9 — 7 — 2 — 4 — 6
    2 — 4 — 9 — 7 — 1 — 6
                    4 — 6 Remain
─────────────────────────
Foot 1 —. —. —. —. —. —. facit.
```

Sixthly, *Of Timber having* 3, 5, 6, 7, *or more ſides equally ſquared.*

If Timber be either Triangular, or elſe conſiſteth of above 4 ſides, you muſt ſtill find the Area of the plane at one of the ends, and then Multiply that by the length, as followeth.

1. *Example.*

Concerning Three ſided Timber.

Suppoſe a piece of Timber 6 foot long to conſiſt of three equal ſides, each ſide being 18 inches, whereof the perpendicular is about 15 inches 5 parts, how many ſolid feet contains it?

To

To find the Area of the plane is to multiply the whole perpendicular by half the baſe or ſide, or the whole baſe or ſide by half the perpendicular, as you have been formerly taught in treating of Triangles.

Operation.

foot inches parts.
1 —— 03 —— 05 perpendicular.

0 —— 07 —— 08 —— 06 } by 9 inches the half of
03 —— 10 —— 03 } 18 the ſide.

· —— 11 —— 06 —— 09 Area of the plane multi-
6 (plyed by the length.

Facit. 5 —— 09 —— 04 —— 06

Note, If all the three ſides are not equal, yet find the Area of the plane as before, and multiply it by the length, or you may multiply the depth by half the breadth, and the product by the length. *Note*, Theſe are called Triangular Priſmes.

2. *Example.*

Concerning Timber that have more than four ſides, called Polygons.

Suppoſe a piece of Timber to have 6 equal ſides, each ſide containing 15 inches, the length 7 foot, how many feet doth it contain?

To find the Area of the plane, the Rule is

Multiply half the perimeter by the whole perpendicular.
The perpendicular is about 13 inches,

The

The Operation.

	foot.	inches.	parts.	
	1	03	00	
			6	
	7	06	00	Perimeter.
	3	09	00	Semi-perimeter.
		3	09	
	4	00	09	Area of the plane.
			7	
Facit	28	05	03	for the whole content.

The like you are to obſerve for the meaſuring of Timber of 5—7—8—9. or more ſides, and this is ſufficient for Timber ſquared.

Seventhly, *Of Round Timber.*

To meaſure round Timber you muſt firſt find the Area of the circle whoſe circumference is equal to the compaſs of the tree you are to meaſure, then multiply that by the length, and you have your deſire.

Now the Area of a Circle may be found either by the diameter, according to the third Problem of circles, or by the circumference according to the 7th. problem of circles.

Note, *round Timber may be beſt meaſured by the feet on the 3 foot Rule, applying them as they are aſſigned.*

L *Exam-*

Example.

A round piece of Timber that is 21 inches diameter and 12 foot long, how many feet contains it ?

```
foot. inches.
  1 —— 09 —— 00 diameter.
  • —— 10 —— 06
        5 —— 03
———————————————
  3 —— 00 —— 09 ſquare of the diameter.
———————————————
       05 —— 03             I work by the diameter
       02 —— 07 —— 06        according to the third
———————————————————          problem of circles.
  • —— 07 —— 10 —— 06
———————————————————
  2 —— 04 —— 10 —— 06 Area of the plain.
                   12
———————————————————
Facit. 28 — 10 —— 06 —— 00
———————————————————
```

Note, There is a common Error crept in amongſt Artificers in meaſuring of round Timber, which is this, *viz.* They take the $\frac{1}{4}$ of the circumference for the ſquare, as in the former piece of round Timber, the circumference is 66, the $\frac{1}{4}$ whereof ($16\frac{1}{2}$) they account the ſquare, which multiplyed in its ſelf produceth $272\frac{1}{4}$ for the Area of the Baſis, which multiplyed by the length the product is 39204 (the content in inches) which divided by 1728, the quotient is $22\frac{2}{3}$ feet, which is apparently erroneous, being above 6 foot too little, as appears before. But they plead there muſt be an allowance for waſt in hewing, which indeed ought to be, but not in falſe meaſuring but rather in the price. Yet if they ſhould bargain for the ſquare Timber

ber that ſhall be contained in it, when ſquared, their way gives not the true ſquare neither, for the ſide of the ſquare is 14 inches, 8 parts $\frac{2}{5}$, whereas it will be found by their way to be 16. $\frac{1}{2}$ inches, ſo that then they pay for more than they have. Therefore if allowance muſt be made as aforeſaid, as in reaſon it ſhould, (the Chips being but worth the labour of Hewing, and in ſome places not ſo much) their exacteſt way will be to agree for ſo much ſquare Timber according to the goodneſs thereof.

Now you may find the ſide of the inſcribed ſquare either by the diameter according to the 8th. problem, or by the circumference according to the 12th. problem of circles, or by the lines on the Rule.

And here for an example, take the round piece of Timber before given, *viz.* 12 inches diameter, which is 66 inches circumference, and 12 foot long, to know how many foot of ſquare Timber is contained in the ſquare thereof.

Operations.

foot. inch. parts.
00—10—06 ſemi-diameter.
2—01— 2— 4
2—01— 2— 4

1—02—08— 4— 8 ſide of the ſquare.
·—02—05— 4— 9—4
·—04—10— 9— 6—8—0
4—10— 9— 6—8—0
4—10— 9—6—8 — 0
4—10—9—6— 8
4—10—9—6— 8

1—06—00— 0— 9—00—5—9— 4 ſquar.
12

Facit. 18—00—00— 9— 0—05—9—4— 0 for the (content

 So

So that it appears when this round piece of Timber is exactly ſquared, it will be 18 foot and a little more.

I have here wrought by the diameter according to the 8th. problem of Circles, multiplying the ſemi-diameter by $1\frac{2}{5}$: And to avoid the trouble of vulgar Fractions, I put them into lower Duodecimals; nor need I have gone farther than the third place, and ſo might have ſaved ſome labour, and yet have come near enough the truth.

Eighthly, *Of Pyramids and Cones.*

If a piece of Timber be right lined, having but one Baſe, and ends in a ſharp point, it is called a Pyramid, but if the Baſe thereof be round it is called a Cone, according to the common Definition.

The ſolid Content of either of theſe, is found by multiplying the ſuperficial Content, or Area of the Baſe by one third part of the length.

Example.

Suppoſe a Pyramid be to be meaſured, whoſe ſide at the Baſe is 18 inches, and the length 45 foot, how many ſolid feet doth it contain?

Reſolution.

Firſt multiply the ſide of the Baſe 18 inches, or 1 foot 6 inches in its ſelf, and the product is the Area of the Baſe, which multiplyed by 15 foot (the third part of the length produceth 33 foot 9 inches, as in the operation.

Opera-

Operation.

foot. inches.
1 —— 06 ſide of the Baſe.
9
——————
2 —— 03 Area of the Baſe.
——————
11 —— 03
——————
Facit. 33 — 09 content of the Pyramid.
——————

Note, *I multiply by the ratio's of* 15, *viz.* 5 *and* 3, *according to the* 8*th.Rule in Multiplication.*

But ſuppoſe the content of a ſegment of this Pyramid were required, the ſide of one end being 18 inches, and the ſide of the other 6 inches, and the length of the ſegment 30 foot, how many feet doth this ſegment contain?

There are two ways given by Authors to perform this problem.

The firſt way.

Find the whole length of the Pyramid by this Analogy.

As the differ. of the 2 ends 12 inch.

	foot.	foot.	foot.	inch.
To the length between them 30 foot.	1 ——	30—	1 ——	6 ?
So is the greater baſe 18 inches		15		
To the whole length 45 foot.		——		
		45. Foot.		

Then find the content of the whole Pyramid, and the content of the leſſer Pyramid, and ſubtract the content of the leſſer from the content of the greater, and the remain is the content of the ſegment.

Thus the whole pyramid being found as before to be 33 fo. 9 inch. and the top thereof 1 foot 3 inch. ſubtracted, there remains for the other part 32 foot and half, and that is the true quantity of the ſegment as in the Margent.

foot.	*inches.*
33 ——	09
1 ——	03
32 ——	06

The

The ſecond way.

Firſt find the Area of each end, then multiply the ſide of the greater end by the ſide of the leſſer, and add the product thereof with the two Areas before found, then multiply the total by $\frac{1}{3}$ part of the length, or the whole length by the $\frac{1}{3}$ part of the total.

Example.

Suppoſe a piece of Timber to have the ſame dimenſions given as before, *viz.* the ſide of one end 18 inches, and of the other 6 inches, and the length 30 foot, how many ſolid feet contains it?

Operation.

```
Foot. inches.                     inches.
 1 —— 06. greater ſide               6 leſſer ſide.
       9                          ————
————  ———                            3 Area thereof.
 2 —— 03 Area thereof.
       3 Area of leſſer ſide.
       9 prod. of the greater ſide mult. by the leſſer.
————  ———
 3 —— 3
      10  the 1/3 part of the length.
————  ———
32 —— 06 the ſame again.
————  ———
```

Note, *Multiplying the ſide of the greater end by the ſide of the leſſer, ſaves the labour of extracting the ſquare root.*

But

But ſuppoſe a piece of Timber bigger at one end than at the other, ſhould not have equal ſides at each end, as ſuppoſe a piece 15 foot long, 8 inches broad, and 4 inches thick at the greater end, and 4 inches broad, and 2 inches thick at the leſſer end, to know how many foot ſolid it contains: To perform this you muſt obſerve this Rule, *viz.*

This is called a Priſmoid.

Firſt find the ſquare or Area of each end, then multiply one Area by the other, and from the product extract the ſquare root, then add the ſquares of the two ends, and the ſquare root ſo found together, and multiply the total by $\frac{1}{3}$ part of the length, and it gives your deſire.

Operation.

Inches.   inches inches.

8 — by 4 inch. greater end   4 by 2 leſſer end.

2 — 8 ſqu. of the greater end   0 — 8 ſqu. thereof.

1 — 4

5 — 4

1 — 9 — 4 prod. of one ſqu. or Area by the other.

inch. parts.

1 — 4 — 0 ſqu. root   2 — 08 ſqu. of great. end

0 — 08 ſqu. of leſſer end.

· — 9 — 4   1 — 04 ſqu. root.

2 — 4

·   ·   4 — 08 total

5 the $\frac{1}{3}$ of the leng.

Facit. 1 — 11 — 04 for the content.

But here obſerve two Cautions.

1. *Caution.*

If in ſquaring the greater end there comes out one foot, or more feet, and but inches in ſquaring the leſſer, you muſt multiply by the inches and parts of the leſſers Area, as if they were feet and inches, and then in extracting the ſquare root you muſt put the feet into inches, adding thereto the inches (if any) and extract the ſquare root of them, and count the ſquare root ſo many inches, &c.

Example.

A piece of Timber 18 inches broad, and 10 inches thick at one end, and 10 inches broad, and 6 inches thick at the other, and 35 foot long, how many ſolid feet doth it contain?

foot inches.
1 — 06. greater end

1 — 03 — 00 ſquare of the greater end.
5

6 — 03 — 00 product.

75

8 — 07 ſquare root.

inches.

inches.
10
5 square of the lesser end.

foot. inches.
1 — 03
5
8 — 07
2 — 04 — 07
11 — 08 the ⅓ part of the (length.
1 — 07 — 00 — 08
26 — 02 — 05
Facit. 27 — 09 — 05 — 08

Note, I multiplyed by 11 foot. 08 inches. according to the second way, of the second Rule in Multiplication.

2. *Caution.*

But if there comes feet in both Areas then you are to work altogether according to the Rule first given.

Example.

As suppose a piece of Timber 27 foot long, 20 inches broad, and 10 inches thick at one end, and 18 inches broad, and 9 inches thick at the other, how many solid feet doth it contain?

foot. inches.
1 — 08
square 1 — 04 — 8 of the greater.
· 1 — 4 — 8
· — 8 — 4
product 1 — 06 — 9 — 0 of one Area by the other.
1 — 03 square Root.
· — 02 — 3
·· — ·

foot.

foot. inches.
1 — 06

ſquare 1 — 01 — 06 of the leſſer end.

foot	inches	parts.
1	04	08
1	01	06
1	03	00
3	09	02
		9 the $\frac{1}{3}$ of the leng.
Facit 33	10	09

Note, The way to perform this by vulgar Arithmetick, is to multiply the Area of one end in inches by the Area of the other end in inches, and to take the ſquare root of the product, and then to add the Areas of the two ends and the ſquare root together, and to multiply their total by the $\frac{1}{3}$ part of the length, and that product to divide by 144, and the quotient gives the content in feet.

Note again, That the common way to work by the ſquare in the middle is very erronious. But yet if the piece be very long and tapering, to avoid the trouble of the ſquare root, you may take your dimenſions in the middle betwixt every 6 foot, and multiply 6 by one ſide, and the product by the other ſide, and when you have ſo gone through the whole length add all together for the whole content; and this will come near enough the truth for common practice.

Again note, That Triangular and Poligonial Pyramids and their ſegments are meaſured the ſame wayes you have been already taught, and therefore there need no Examples of either.

Ninthly,

Ninthly. *Of Globes.*

Problem 1.

To find the Solid Content of a Globe.

Multiply the Cube of the Diameter by 11, and divide the product by 21, and the quotient is the ſolid content.

Example.

Suppoſe a Globe to be meaſured whoſe Diameter is 21 inches, how many ſolid feet doth it contain?

	foot.	inches.			
	1	09	00		diameter.
	1	03	09		
	3	00	09		
	2	03	06	9	
Cube	5	04	03	9	
				11	
21)	58	11	05	03	
7)	8	05	00	09	
3)	2	09	08	03	*Facit.*

Note, *If you take the* $\frac{1}{3}$ *of the Cube of the Diameter the* $\frac{1}{2}$ *of that* $\frac{1}{3}$ *and the* $\frac{1}{7}$ *of that* $\frac{1}{2}$*, and add them together, it will give the content of the Globe.*

In cubing the Diameter I multiplyed according to the ſecond way of the ſecond Rule in Multiplication. And in dividing by 21, I divided by the ratio's thereof, *viz.* 7 and 3 according to the 4th. Rule in Diviſion.

Prob.

Problem 2.

Having the Content of a Globe to find its Diameter.

Multiply the content by 21, and divide it by 11, and the Cube root of the quotient is the Diameter.

Example.

There is a Globe whoſe Content is 2—09—08—03 foot, I demand its diameter ?

	foot.				
	2	09	08	03	
	19	07	09	09	I multiplyed by the ratio's of 21, *viz.* 7 and 3.
11)	58	11	05	03	
	5	04	03	09	Quotient.
Facit -	1	09	··	··	Cube-Root.

Problem 3.

Having the Diameter of the Baſe of the Fruſtum of a Globe (intercepted between two planes parallel, one touching the other cutting the Globe) together with the Fruſtums Altitude, to find the Globes Axe.

Rule.

Divide the ſquare of the Semi-Diameter of the Fruſtums Baſe by the Altitude of the Fruſtum, and add the Altitude of the Fruſtum to the Quotient, and the total gives the Globes Axe.

Exam-

Example.

Let the diameter of the Fruſtums Baſe of a Globe be 16 inches, and the Altitude of the Fruſtum 4 inches, I demand the Globes Axe.

Note, *To divide by* 4 *inches, I ſay* 4 *inches is* $\frac{1}{3}$ *and therefore I multiply by* 3 *and the product is the quotient.*

inches.
8 —— ſemi-diameter
———— ————
4
1 —— 4
———— ————
$\frac{1}{3}$) 5 —— 4 ſquared.
———— ————
1 —— 4 —— 0 quotient.
4 —— 0 altitud. of the fruſtum.
———— ————
1 —— 8 —— 0 for the Axe.
———— ————

Again let the greater ſegment or Fruſtum be given, *viz.* The Baſe 16, and the Altitude 16, I demand the Axe?

inches.
8 —ſemi-diameter.
———— ————
4
1 —— 4
———— ————
1 —— 4) 5 —— 4 (4 quotient.
5 —— 4 16 fruſtum Altitude.
———— ————
· —— · 20 inches or 1 foot 8 inches.
————

The ſame again.

Prob.

Problem 4.

Having the Altitude of the Fruſtum, and the Globes Axe, to find the Diameter of the Fruſtums Baſe.

The Rule.

A Geometrical mean proportion between the greater and leſſer Fruſtums Altitudes is equal to the Semi-diameter of the Fruſtums Baſe, which doubled is the diameter of the Baſe required.

Example.

Let the Altitude of the fruſtum be the ſame with the former, *viz.* 4 inches, and the greater fruſtums Altitude 16 inches, I demand the fruſtums Baſe.

$$\begin{array}{rl} & 16 \\ & 4 \\ \hline & 64 \\ \hline \text{Doubled} & \left\{\begin{array}{l} 8 \text{ ſquare root.} \\ 8 \end{array}\right. \\ \hline & 16 \text{ the diameter of the fruſtums Baſe.} \\ \hline \end{array}$$

Problem 5.

Having the Globes Axe, the Segments Altitude, and the Diameter of its Baſe to find the Segments ſolid Content.

The Rule.

Multiply the ſquare of the ſegments Diameter or Baſe by the ſegments Altitude, and the product will be a Parallelopipedon, which product keep.

Then

Then as the Altitude of the other ſegment is to $\frac{1}{2}$ the ſame Altitude, and more $\frac{1}{6}$ of the given Altitude, ſo is the Parallelopipedon firſt found to all the ſquares in the ſegment.

Laſtly, As 14 to 11, ſo are all the ſquares laſt found to the ſolid content of the ſegment required.

Example.

Let the Globes Axe be 3 foot, 4 inches. The Altitude of the ſegment 10 inches, and the ſegments Baſe 2 foot, 10 inches 07—09. I demand the ſegments content?

Foot.
2 — 10 — 07 — 09 ſegments diameter.
2 — 10 — 07 — 09
2 — 04 — 10 — 05 — 06
· 1 — 08 — 02 — 06 — 03
· 2 — 01 — 11 — 09 — 9

Foot. 8 — 04 — 00 — 04 — 00 — 00 — 9 ſqua.
10

6 — 11 — 04 — 03 — 04 — 00 — 07 — 06 Pa-(rallelopipedon.

foot. inches.
2 — 06 the other ſegment.

1 — 03
01 — 08 the $\frac{1}{6}$ of the given Altitude.

1 — 04 — 08 Now, I ſay

If

fo. inch. foot. foot.

If 2-06 give 1-04-08 : what 6—11—04—03
2—03—09—05
02—03—09—5
02—03—09—5

2—6) 9—07—09—02—10—(3—10
7—06

2—01—09
2— 1—00

·—00—09

foot.

If 14 give 11 : what 3—10

14) 42— 2

6— 0—3—5

Facit 3— 0—1— 8—6 very near, but ſomewhat too little by omitting parts.

Inſtead of multiplying by 11 and dividing by 14 you may take the $\frac{1}{7}$ and $\frac{1}{2}$ of that, and ſubtract their total.

Problem 6.

But more brief this may be performed thus, without the knowledge of the Fruſtums Diameter of the Baſe, *viz.*

1. From the Globes Axe ſubduct the fruſtums Altitude, and unto the remainer adde the Globes ſemi-axe.

2. Multiply that ſum by the ſquare of the fruſtums Altitude.

3. Multiply that product by 1 foot 00 – 7, and it produceth the content of the fruſtum required.

And

And here for an Example take the ſame again, *viz.*

Let the Globes Axe be 3 foot,04 inches, and the fruſtums Altitude 10 inches. I demand the content of the Fruſtum ?

foot. inches.
3 — 04. The Globes Axe.
10 The ſegments Altitude.

2 — 06 The greater Fruſtums Altitude.
1 — 08 The Globes ſemi-Axe.

4 — 02 Their ſumme.
8 — 4 ſquare of the Altitude given.

2 — 09 — 4
· — 01 — 4 — 8

foot.
2 — 10 — 8 — 8 multipl. by 1—00—07
· — 01 — 8 — 3 — 0 — 8

Facit 3 — 00 — 4 — 11 — 0 — 8 for the ſolid Content of the ſegment, which comes very near the truth, onely a little too much, but ſo little as is very inconſiderable ?

Problem 7.

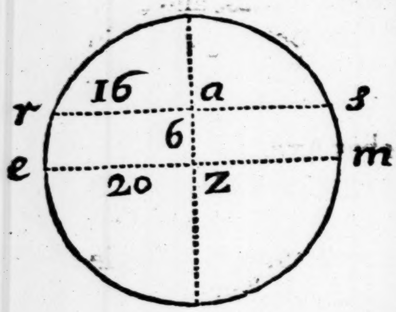
Having the Altitude of a Globes Fruſtum, with the Diameter of its Baſe, to find the Fruſtums Content without the knowledge of the Globe Axe.

The Rule.

Firſt find the Globes Axe, by the Rule for Problem the Third.

Secondly, Then work by the Rule for Problem the Fifth.

Problem 8.



Having the Axe of a Globe or Sphere *e m*, 20 inches and *z a*, 6 inches (the Altitude of the Zone of the Globe *r e ſ m z a*) together with the diameter of its leſſer Baſe *r s* 16 inches, to find the Zones ſolid Content.

The Rule.

Firſt find $\frac{2}{3}$ of the Area of the greater Baſe, and $\frac{1}{3}$ of the Area of the leſſer Baſe, by the third Problems of Circles.

Secondly, Having added them together, multiply their ſumm by the Zones Altitude, and that product gives your deſire.

Operation.

greater diameter 1 — 08
06 — 8
06 — 8

ſquared — 2 — 09 — 4

$\frac{1}{7}$ — · — 04 — 9 — 1 — 8
$\frac{1}{2}$ — — 02 — 4 — 6 — 10

ſumme deducted — 07 — 1 — 8 — 06

Area 2 — 02 — 2 — 3 — 06

$\frac{2}{3}$ — · — 08 — 8 — 9 — 02 } of greater
— · — 08 — 8 — 9 — 02 } Area.
$\frac{1}{3}$ — — 05 — 7 — 0 — 07 of leſſer Area.

1 — 11 — 0 — 6 — 11 ſumme.

Altitude 6 inch. 0 — 11 — 6 — 3 — 05 — 06 *facit.*
foot.

foot. inches.
Leſſer diameter 1 —— 04
05 —— 4

ſquared 1 —— 09 —— 4

$\frac{1}{7}$ —— · —— 03 —— 0 —— 6 —— 10
$\frac{1}{2}$ —— —— 01 —— 6 —— 3 —— 05

ſumme deduct. —— 04 —— 6 —— 10 —— 03

Area 1 —— 04 —— 9 —— 01 —— 09

$\frac{1}{3}$ —— · —— 05 —— 7 —— 00 —— 07

Or, *Thus more brief*, viz.

To the double of the Area of the greater Diameter, add the Area of the leſſer, and multiply their ſumme by $\frac{1}{3}$ of the Zones Altitude.

foot.
Area of the greater diameter 2—2—2—3—6
2—2—2—3—6
1—4—9—1—9

5—9—1—8—9

The $\frac{1}{3}$ of 6 inch. (the Zones Alt.) is 2 inch. which is the $\frac{1}{6}$ —— · —— 11 —— 6 —— 3 —— 5 —— 6 The ſame again.

Problem. 9.

There is a Globe whoſe ſolid content is 2 foot, 9 inches, and a Cube is deſired to be made that ſhall be of the ſame content with the Globe, the Queſtion is what length the ſide of the Cube muſt be? And alſo the length of the Globes Axe.

The Rule.

Firſt, To find the ſide of the Cube, the Cubick Root of the Globes ſolidity is the ſide of Cube that ſhall be of the ſame content with the Globe.

Secondly, To find the Globes Axe, it is performed by the ſecond Problem of Globes.

Duo-

Duodecimal
GAUGING.

BOOK. III.

Shewing how to Gauge all ſorts of Brewers Tuns and Casks by Duodecimal Arithmetick, all being practically performed, viz. *By the Aliquot parts of* 12.

FOR the better Application of Duodecimal Arithmetick to the Art of Gauging, I have thought on a way to perform it onely by Duodecimal Multiplication, which is indeed no other than the Common Rule of Practice, for which purpoſe, I have made the enſuing meaſures, which muſt all be Duodecimally divided, *viz.* into primes, ſeconds, *&c.*

Firſt, I have made a meaſure, the length of the ſide of a ſquare that ſhall contain 282 inches, (the number of cubical

inches contained in an Ale Gallon,) and the like for 231 inches, (the number of cubical inches contained in a Wine Gallon,)the one being the ſquare root of 282, and the other the ſquare root of 231, which I call ſuperficial or ſquare Gallons.

Now by this meaſure you may find the ſuperficial content at one inch deep, of any Veſſel bounded with right lines, which multiplyed by the depth in inches, (provided there be the ſame dimenſions at top and bottom) you will have the whole ſolid content in Ale or Wine Gallons. As for the uſe of it in Gauging Veſſels bounded with right lines differing in their dimenſions at the top and bottom, I ſhall ſhew when I come to treat of them.

Secondly, I have made another meaſure both for Ale and Wine which I call the circle ſquare, that for Ale, being the ſide of a ſquare equal to the diameter of that Circle whoſe Area is 282 inches, and that for Wine, the ſide of a ſquare equal to the diameter of that Circle whoſe Area is 231, both being the ſquare root of the gauge numbers reſpectively, ſo that the Area of any Circle in Ale or Wine gallons, may be found, only by ſquaring the diameter taken by this Meaſure.

Thirdly, I have made another meaſure both for Ale and Wine, which I call the Circles $\frac{2}{3}$ ſquare; being the ſquare root of the gauge numbers reſpectively of the ſumme of the inches, and $\frac{1}{3}$ thereof contained in the Ale and Wine gallons, ſo that the $\frac{2}{3}$ Area of any Circle in Ale or Wine gallons, may be found only by ſquaring the diameter taken by this meaſure.

Fourthly, I have made another meaſure both for Ale and Wine, which I call the Circle $\frac{1}{3}$ ſquare; being the ſquare root of the gauge numbers reſpectively of the ſum of the inches and $\frac{2}{3}$ thereof, contained in the Ale and wine gallons, ſo that the $\frac{1}{3}$ Area of any Circle in Ale or Wine gallons may

may alſo be found by ſquaring the diameter, taken by this meaſure.

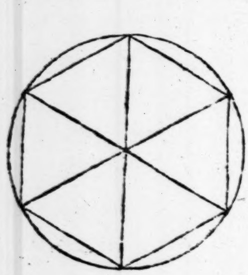
Fifthly, In regard that it often happens that the diameters cannot be taken (as of ſtanding Casks, &c.) but muſt be found by the circumference, by girting. I have made a meaſure both for Ale and Wine, which taken on the Circumference, and ſquared, gives the Area of the Circle, which I call the circumference ſquare; and is thus made. *viz.* The diameter being $\frac{7}{11}$ of the circumference, as 7 is to 1 Circle ſquare gallon, ſo will 4 be to that part of the ſame gallon, which muſt be added to it, for the length of one gallon on the circumference: And here note, having found the gallons on the circumference, by this meaſure, before you ſquare it, you muſt allow for the thickneſs of the Staves at each end of the diameter according to diſcretion, which allowance muſt be taken out of the ſame number of gallons on the circle ſquare, and then ſquare the remain; for the ſame number of gallons found on the circumference, would have been found on the diameter, if you could have come at it.

Sixthly, For the eaſe of Countrey Officers, I have made a meaſure both for Ale and Wine, which I call the Circle gauge, which lain on the diameter of any Circle, whoſe Area exceed not 15 gallons, will give the area or ſuperficial Content of that Circle without ſquaring, or any thing more to be done; being the ſquare root of the gauge number of every gallon ſucceſſively.

Seventhly, I have made another meaſure both for Ale and Wine, which I call the Globe Cube; which lain on the Axis of any Sphere or Globe; and the number found thereby, cubed, ſhall give the ſolid content in gallons, whether of Ale or Wine reſpectively, and this meaſure is thus made, *viz.* as 11 is to 21, ſo will the ſolid content of any Globe or Sphere be to that number, the Cube root whereof

will be the diameter, therefore $\frac{11}{21}$ being $\frac{10}{21}$ less than unity, if you add $\frac{10}{21}$ of the number of inches contained either in an Ale or Wine gallon unto that number ; the Cube-root of the summe will be the side of the Cube, that shall be equal to the Axis of that Sphere or Globe, whose solid content is 1 gallon, whether of Ale or Wine respectively.

I have contrived all these measures to be carryed in an underhand Cane, and yet to be made to measure Dimensions (if need require) of 18 foot at once taking. The manner whereof (if you will have it of that length,) you may see in the Margent, whereby you may see that the Scale or Ruler will then be Triangular, divided into 6 parts (and so a Hexagon) which must be joyned together with Sockets, and so the Cane being 3 foot the Scale or Ruler will be 18 ; but you may have it made either of what length or form you please.



On one part of the scale or ruler must be one of each measure, duodecimally divided, continuing the divisions throughout the part, every measure corresponding with his own proper measure on the rest of the parts, and a line of inches must be on the same part duodecimally divided thorowout, and this part must slide in 2 sockets, to take the over measure.

The Length of the Measures.

		inches	′	″	‴		
1. The superficial or square gallon for	Ale	16	9	6	2		
	Wine	15	2	4	7	4	
2. The Circle square gallon for	Ale	18	11	4	4	9	
	Wine	17	01	9	1	0	3

3. The

		inch. ′	″	‴	⁗	′′′′′
3. The Circles : squ. for-	Ale	21	10	6	1	2 — 2
	Wine	19	09	7	0	7
4. The Circles ; squ. for-	Ale	24	05	5	11	0
	Wine	22	01	7	06	10

		inch.	′	″	‴	⁗	
5. The Circumference square for	Ale	1	6	10	3	5	of the Circle squares respectively
	Wine	1	6	10	3	5	

6. The Circle Gauge, for- { Ale— / Wine } both to be made by the Tables, Calculated for that purpose.

7. The Globe Cube for { Ale— 7 inch. 5—7—3 / Wine 6—11—10—0 }

Note, *The Officer that is imployed in the Excise for Beer and Ale, will have occasion but for* 4 *lines to be put on his Rule, besides a line of Inches,* viz.

1. The Superficial square Gallon
2. The Circle square Gallon
3. The Circumference squ. Gallon
8. The Circle Gauge

} For Ale.

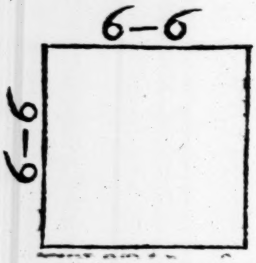
I. *Of Vessels bounded with Right Lines.*

First, FOR here I shall take the same method that I took in treating of Superficies and Solids, and so shall begin with Vessels that are square solids, having the same dimensions at top and bottom.

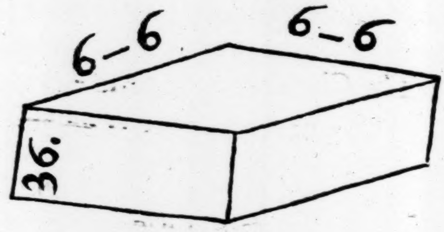
1. *Ex-*

1. Example.

Superficial.



Solid.



A Tun 6 gallons 6 primes ſuperficial, ſquare at top and bottom, and 36 inches deep, how many Beer barrels, and how many gallons doth it contain?

And how many gallons on each wet inch?

The Rule.

Multiply one ſide into it ſelf (that is the length by the breadth) in gallons and primes, and the product by the depth in inches.

Note, you muſt always multiply by the ratio's of the number of the inches of the depth, which is here 36, and the ratio's are 6 and 6.

Operation.

gallons.
6 —— 6 the ſide.

39 —— 0
3 —— 3

gallons on each wet inch 42 —— 3 barrels in all.

253 —— 6

gallons in all 1521 —— 0

Note,

Note, Having multiplyed by 36, (which is the number of gallons in a Beer barrel,) to bring the gallons into barrels, I must have divided by 36, and so the quotient would have produced the same multiplicand again, therefore I conclude the Tun contains 42 —— 3′ barrels and 1521 gallons.

1 . *Example.*

A Tun $\overset{\text{gallons}}{8} — \overset{\prime}{9} — \overset{\prime\prime}{6}$ square at top and bottom, and 48 inches deep, how many gallons, and how many barrels is the whole Tuns capacity?

gall.
8—09—06

70—04—00
4—04—09
2—02—04—06
4—04—09

77—03—06—03 gall. on each wet (inch

12—10—07—00—6 } divided by the ratio's
bar. on each wet inch 2—01—09—02—1 } of 36, *viz.* 6 and 6.

17—02—01—04—8 } multiplyed by the ra-
barrel in all 103—00—08—04—.. } tio's of 48 *viz.* 8 and 6

So that the Tun contains on each wet inch $\overset{\text{gall.}}{77} - \overset{\prime}{3} - \overset{\prime\prime}{6} - \overset{\prime\prime\prime}{3}$, which makes $\overset{\text{bar.}}{2} — \overset{\prime}{1} — \overset{\prime\prime}{9} — \overset{\prime\prime\prime}{2} — \overset{\prime\prime\prime\prime}{1}$: That is to say, 2 barrels, 5 gallons, 2 pints $\frac{2}{12}$ of a pint, and $\frac{1}{12}$ of the $\frac{1}{12}$ of a pint: for $\frac{1}{12}$ of a barrel is 3 gallons, and the $\frac{1}{12}$ of 3 gallons is 2 pints, *&c.*

And it contains in all 103 bar. 2 gall. and $\frac{2}{3}$ of a pint.

Note,

Note, To bring gallons into barrels I alwayes divide by the ratio's of 36 for Beer, *viz.* 6 and 6, and by the ratio's of 32 for Ale, *viz.* 8 and 4.

Note again, If you take the $\frac{1}{6}$ of each ſide, and multiply them one by the other the product will be the Beer barrels contained on one inch which multiplyed by the depth in inches will be the Beer barrels in all.

So if you take the $\frac{1}{8}$ of one ſide, and the $\frac{1}{4}$ of the other, and multiply them one by the other, the product will be the Ale barrels contained on one inch, &c.

II. *Of Veſſels that are not Square.*

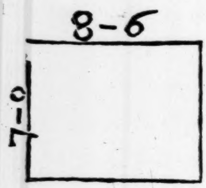
UNder this Head are comprehended Veſſels, that are

1. Longer than broad.
2. Broader at one end, than at the other.

First, *Of Veſſels that are longer than broad.*

The Rule is the ſame with the foregoing, that is to ſay, to multiply the length by the breadth, and the product by the depth in inches.

1. *Example.*



A Tuns length is 8 gallons, 6 primes, the breadth 7 gallons at top and bottom, the depth 56 inches, how many gallons, and how many barrels doth each wet inch contain? And how many barrels is the whole Tuns capacity?

Gallon

gallons.
8 —— 06

gall. on each wet inch 59 —— 06

9 —— 11

barrel on each wet inch 1 —— 07 —— 10

13 —— 02 —— 01

barrels in all — 92 —— 06 —— 08 that is to ſay 92 bar. $\frac{1}{2}$ and 2 gallons

Note, It may ſometimes happen in long Tuns, that there may be a difference between the two ends, and another between each end and the middle: in ſuch caſes you muſt take an Arithmetical mean between the two ends, and another between that mean and the middle; then will the laſt mean reduce the Tun into a perfect Oblong. Some do add the two ends and the middle together, and divide their ſum by 3, but that is very erroneous.

2. *Example.*

A Tun 11 gall. — 10′ long, 11 gall. — 02′ — 06″ broad at top and bottom, and 60 inches $\frac{1}{2}$ in depth, how many gallons and how many barrels doth each wet inch contain? And how many barrels contains the whole Tun?

gallons

	gallons.
	11—10 long
Multiplied by the breadth.	130—02
	1—11— 8
	05—11
On each wet inch.	gallons 132—07—07
	22—01—03—2
	barrels 3—08—02—6-4
div. by rati. of 36.	
Multiplyed by the ratio's of 60, *viz.* 12 and 5, to which I add the ½ for ½ an inch.	44—02—06—4—0
	221—00—07—8—0
	1—10—01—3—2
barrels in all.	222—10—08-11—2

3. *Example.*

A Cooler $9^{gall.}$—$10^{\prime}$—$09^{\prime\prime}$ broad, and $10^{gall.}$—$06^{\prime}$—$04^{\prime\prime}$ long, and 20 inches and half deep, how many gallons and how many Beer barrels may each wet inch contain? And what is the Coolers whole capacity in barrels?

gallons

gallons , ″
9—10—09

98—11—06
4—11—04—06
03—03—07

On each wet inch.	gall. 104—02—02—01	div. by the rati. of 36.
	17—04—04—04—2	
	barr. · 2—10—08—08—08—04	

Multiplyed by the ratio's of 20, *viz.* 5 and 4, to which I added the half for half an inch.	14—05—07—07—05—08
	57—10—06—05—10—08
	1—05—04—04—04—02
barrels in all	59—03—10—10—02—10,

viz. 59 barrels, 11 gallons and almost $\frac{3}{4}$,

For the Dripp or fall take this general Rule, *viz.*

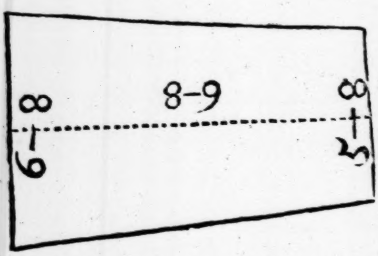
Into the Tun proposed (being empty) power liquor until the uppermost part of the bottom be just covered, then find the greatest depth of the liquor, and half that depth is the Tuns fall or dripp, which subducted from the Tuns depth leaveth the Vessels true depth to be reckoned on, and made use of in inching the Tun.

But I esteem it the exactest way to measure the liquor poured in, because there may be several different and irregular falls in the bottom, then you may gauge it from the superficies of the liquor and add the liquor thereunto at every inch, &c.

Se-

Secondly, *Of Vessels broader at one end, than at the other, the ends and bases parallel, called frustums of Prisms.*

Example.



A Tun 8 gallons 9 primes long, 6 gall. 08 primes broad at one end, and 5 gallons and 8 primes broad at the other, and 40 inches deep, (of the same dimensions at top and bottom, and ends and bases parallel,) how many gallons, and how many barrels, may each wet inch contain, and what is the Tuns whole capacity in barrels?

The Rule.

Add the two breadths together and take the ½ thereof for the mean breadth, and thereby multiply the length.

gallons ,
6 — 8 greater breadth.
5 — 4 lesser.

½ — 12 — 0

mean breadth — 6 — 0

gall. 8 — 9 long

on each wet inch — { gall. 52 — 6
8 — 9
barr. 1 — 5 — 6 } divided by the ratio's of 36.

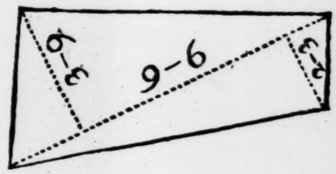
11 — 8 — 0 } Multiplyed by the ratio's of 40, *viz.* 8 and 5.

barrels in all — 58 — 4 — 0

Thirdly,

III. *Of Vessels in the form of a Trapezial Solid.*

Example.



A Trapezial solid Tun of the same dimensions at the top& bottom, whose longest diagonal is 9 gallons 6 primes, one perpendicular 2 gallons 3 primes: and the other 3 gallons 09 primes, and 56 inches deep: how many gallons, and how many barrels may each wet inch contain? And how many barrels contains the whole Tun?

The Rule.

Add both Perpendiculars together, and by half of their total, multiply the whole Diagonal, or multiply half the Diagonal by the whole total.

gall. ′
2 —— 3 } Perpendiculars.
3 —— 9 }

6 —— 0

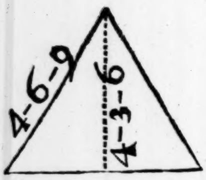
The half 3 —— 0
gall. 9 —— 06 Diagonal

Per wet inch, gallons 28 —— 06 } multiplied by the ratio's of 56.
228 —— 00 }

The Tuns content in { gallons 1596 —— 00 } divid. by the ratio's of 36.
266 —— 00
{ barrels 44 —— 04 } *viz.* 44. barr. (12 gallons.

IV. *Of Triangular Solid Tuns.*

1. *Example.*



An Equilateral Triangular solid Tun, whose side is 4 gall. 6 primes, 9 seconds: its perpendicular about 4 gall. 3 primes 6 seconds, the depth 54 inches: how many gallons may each wet inch contain, and what is the Tuns whole capacity in gallons and barrels?

The Rule.

Multiply the whole side by half the perpendicular, or whole perpendicular by half the side.

gall. ′ ″
4-03-06 perpendic.
2-01-09 the half.

gall. ′ ″
4-06-09 the side

9-01-06
04-06-09
02-03-04-06
1-01-08-03 } Multiplyed by half the perpendicular.

gallons on each wet inch 9-09-05-09-09

88-01-04-03-09 } Multiplyed by the ratio's of 54, *viz.* 9 and 6.

The Tuns capacity in — { gallons 528-08-01-10-06

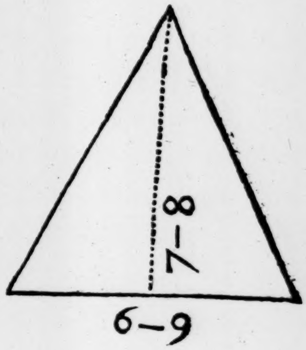
88-01-04-03-09

barrels 14-08-02-08-07-06 } divided by the ratio's of 36.

2. *Ex-*

2. *Example.*

A Tun in the form of an Isosceles Triangular solid, whose base is 6 gallons, 09 primes, and the perpendicular 7 gallons, 08 primes, the depth 64 inches (the top and bottom of the same dimensions,) how many gallons may each wet inch contain, and what may be the content of the whole Tun in gallons and barrels?

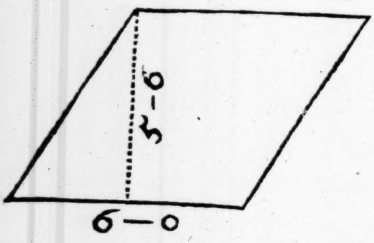


gall.	′		gall.	′	
7 —	08 perpendicu.		6 —	— 09 the base	
3 —	10 the half.		20 —	— 03	
			3 —	04 —	06
			2 —	03 —	00
On each wet inch gall.			25 —	10 —	06
			207 —	00 —	00
The whole Tun contains—	gallons —		1656 —	00 —	00
			276 —	00 —	00
	barrels —		46 —	00 —	00

Note, The same rule holds for all other Triangular solids, *viz.* to multiply the base by half the perpendicular, or half the base by the whole perpendicular.

V. *Of Tuns in the form of a solid Rombus or Romboides.*

First, *Of a Rombus.*



There is a Tun in the form of a solid Rombus (the top and bottom of the same dimensions,) whose 4 sides are each 6 gallons, the perpendicular 5 gallons, 06 primes and the depth 64 inches? how many gallons contains each wet inch, and how many gallons, and how many barrels contains the whole Tun?

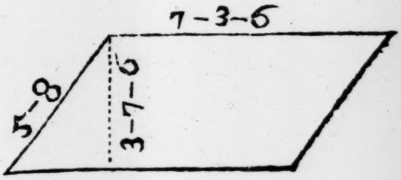
The Rule for both.

Multiply one side by the Perpendicular,

	gall. ′	
	6 —— 00	one side
	30 —— 00	multiplyed by the
	3 —— 00	perpendicular.
gallons on each wet inch	33 —— 00	
	264	mult. by the rat. of 64, *viz.* 8&8.
The Tun contains in all —— gall.	2112	
	352	divided by the ratio's of 36.
barr.	58 —— 08	*viz.* 58 bar. 24 gall.

Secondly,

Secondly, *Of a Romboides.*



A Tun in the form of a ſolid Romboides (the top and bottom of the ſame dimenſions,) the longeſt ſides are 7 gallons, 3 primes, and 6 ſeconds, the ſhorteſt are 5 gall. 8 primes, the perpendicular 3 gallons 07 primes, 6 ſeconds, and the depth 58 inches, how many gallons may each wet inch contain? and what is the Tuns whole capacity in gall. and barrels?

	gall. ′ ″	
	7-03-06 longeſt ſide.	
	21-10-06	mult. by the perp.
	4-03-00-06	
	03-07-09	
On each wet inch gall.	26-05-02-03	
	211-05-06-00	mult. by the ratio's of 56, *viz.* 8 & 7, & to the laſt rat. prod. the product of 2 added.
	1480-02-06-00	
	52-10-04-06	
The whole Tuns capacity in — gallons	1533-00-10-06	divid. by the ratio's of 36.
	255-06-01-09	
barrels	42-07-00-03-06	*viz.* 42 barr. 21 gall. and ½ a pint and ſomewhat more.

A Rule to reduce Beer Barrels into Ale, or Ale into Beer.

Note, 32 gallons makes a barrel of Ale, and 36 a barrel of Beer, so that to reduce any number of Beer barrels to Ale, or Ale to Beer, the proportion will be as 9 is to the Beer barrels, so is 8 to the Ale, and as 8 is to the Ale so is 9 to the Beer in a reverse proportion. And instead of multiplying by 8 and dividing by 9, take the $\frac{1}{9}$ and subtract it; and in stead of multiplying by 9 and dividing by 8, take the $\frac{1}{8}$ and add it, according to the 7th. Rule in Division.

Of Reducing Ale barrels to Beer barrels.

Example.

In 58 barr. 08 primes of Ale, how many barrels of Beer.

Bar. ′
58 —— 8 I take the $\frac{1}{9}$ and subtract it.
6 —— 6 —— $\frac{2}{9}$

Facit. 52 —— 1 —— $\frac{7}{9}$ barrels of Beer.

Of Reducing Beer barrels into Ale.

Example.

In 52 bar. 01 — $\frac{7}{9}$ of Beer, how many barrels of Ale.

52 —— 01′ — $\frac{7}{9}$ I take $\frac{1}{8}$ and add it.
6 —— 06 — $\frac{2}{9}$

Facit — 58 —— 08 — 0 barr. of Ale.

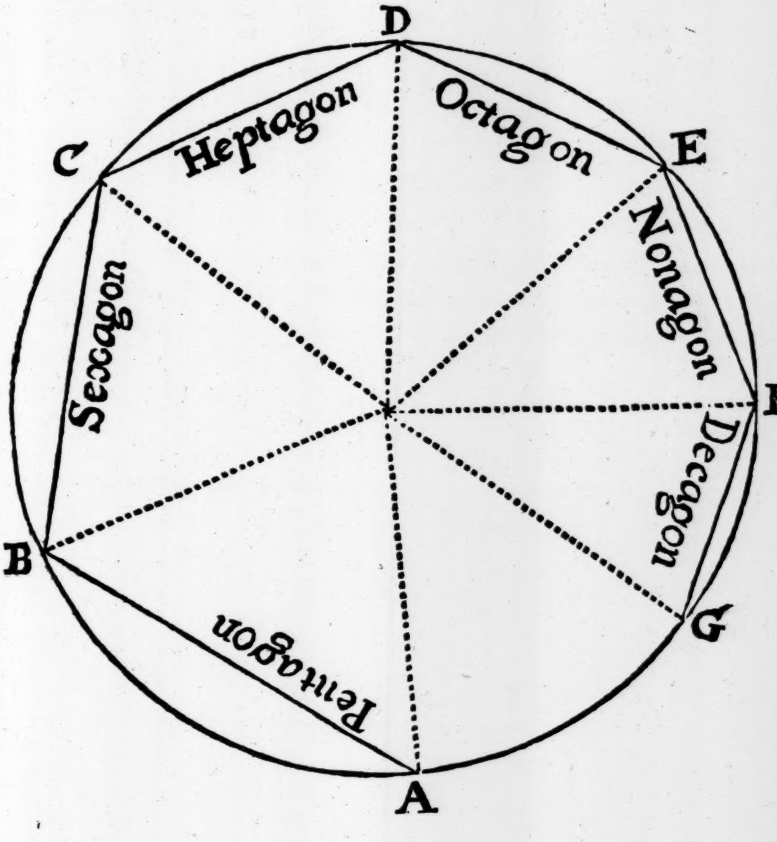
VI. *Of*

VI. *Of Tuns in the form of Poligonial Solids.*

To measure a Regular Poligon.

The Rule is,

To multiply $\frac{1}{2}$ the perimeter (*viz.* half the ſum of the ſides) by the line drawn perpendicular from the Center, to the middle of one of the ſides.



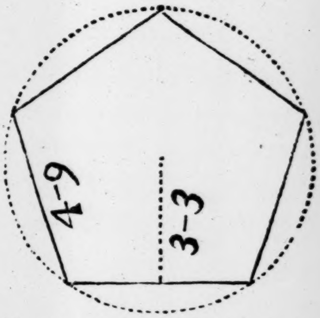
And for the finding the perpendicular, you may make an instrument by describing a circle, and dividing it into 5 equal parts for a Pentagon, into 6 for a Sexagon, into 7 for a Heptagon, into 8 for an Octagon, into 9 for a Nonagon, into 10 for a Decagon, *&c.* and lay them down on another circle of the same diameter, as you may in the foregoing circle see it done, as from *A* to *B* for a Pentagon, from B to *C* for a Sexagon, from *C* to *D* for a Heptagon, from *D* to *E* for an Octagon, from *E* to *F* for a Nonagon, from *F* to *G* for a Decagon, and it may be made on brass or what you please. Then having one of the sides, take it between your Compasses from a scale of equal parts and lay it down on the circle, between its proper rays parallel to the side, *viz.* if a Pentagon between the rayes of the Pentagon, *&c.* then may you take the perpendicular in your Compasses, *viz.* the nearest place between the Center and the side so layd down, and find it by the same scale of equal parts.

Or he that will be at the charge of a pair of dividing Compasses (made by * Mr. *John Oldum* in *Red-Cross-Court* in *Tower-street, London*) may place the joynt or nut on the number of sides, and then extend the dividing points on a small scale of equal parts to the length of 1 of the sides, then will the other points be extended to the length of the semi-diameter of that circle whereof the Poligon is made, then may he draw an Arch at that distance with another pair of Compasses, and lay thereon the side, and the nearest distance between the center and that side will be the perpendicular.

* *He maketh all sorts of other Compasses in Brass, Steel or Silver, and other Mathematical instruments in brass.*

1. *Ex-*

1. *Example.*



A Tun in the form of a Pentagonial ſolid, (*viz.* 5 ſides) each ſide 4 gallon 09 primes, and the perpendicular line drawn from the center to one of the ſides 3 gallons 03 primes, and the depth 58 inches (the dimenſions at top and bottom the ſame,) how many gallons and barrels may each wet inch contain? And how many barrels contains the whole Tun?

gall. ′ ″
2-04-06 the half of one ſide.
5 the number of ſides.

Note, inſtead of dividing by 36, you may multiply by 0—0′—4″

11-10-06 the ſemi-perimeter.

35-07-06
2-11-07-06 } multiplied by the perpendicular.

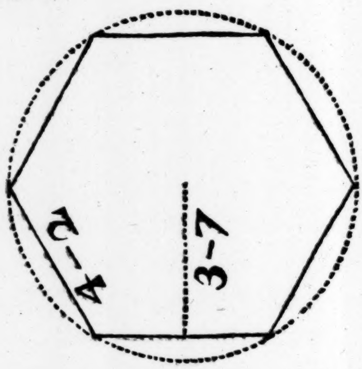
On each wet inch are— { gallons - 38-07-01-06
6-05-02-03
barrels. 1-00-10-04-06 } divided by the ratio's of 36.

8-06-11-00-00
60-00-05-00-00
2-01-08-09-00
Barrels in all— 62-02-01-09-00 } multipl. by the ratio's of 56, *viz.* 8 and 7 to to which I added the product of 2.

2. *Ex-*

2. *Example.*

A Tun in the form of a Hexagonial ſolid (having 6 equal ſides) each ſide being 4 gallons 02 primes, and the line drawn perpendicular from the center to one of the ſides is 3 gallons 07 primes, the depth 56 and half inches, (the top and bottom of the ſame dimenſions,) how many gallons and barrels may each wet inch contain? And how many barrels is the Tuns whole capacity?



gall. ′
2—01 the half of one ſide.
6 the number of ſides.

12—06 ſemi-perimeter.
3—07 perpendicular.

7—03—06
37—06

On each wet inch are — gallons - 44—09—06 product.

7—05—07 div.by rati.of 36.

barrels. 1—02—11—2

9—11—05—4

69—08—01—4
07—05—7

barrels in all 70—03—06-11

multiplyed by the ratio's of 56, and the product of half an inch added.

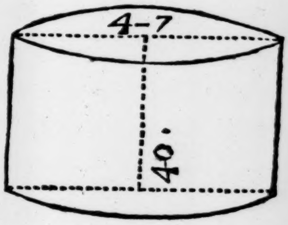
The

The ſame Rule is to be obſerved for all other Polygonial ſolids, and therefore more Examples would be needleſs.

And as for Gauging of Veſſels of many ſided irregular forms, they are to be reduced into Triangles or Trapezia's, by drawing of Diagonal lines from one corner to another, and then meaſured, and caſt up ſeverally, according to former Rules, therefore Examples of this nature would be needleſs alſo.

II. *Of Veſſels bounded with Lines that are Circular, having the ſame dimenſions at top and bottom.*

First, A Cylindrical Tun whoſe diameter at each baſe is 4 gallons, and 7 primes, of the circle ſquare, and depth 40 inch. how many gallons and Beer barrels may each wet inch contain, and how many barrels contains the Tun in all?

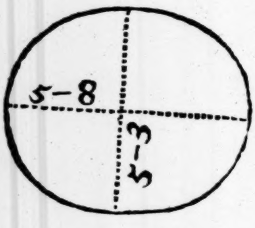


The Rule.

Square the Diameter, and multiply the product by the inches of the depth.

gallons.

	gall. ′	
	4—7	
	4—7	
	18—4	
	2—3—6	
	4—7	
On each wet inch — gallons	21—0—1	divided by the ratio's of 36.
	3—6—0—2	
barrels	0—7—0—0—4	
	4—8—0—2—8	multipl. by the ratio's of 40, *viz.* 8 and 5.
barrels in all	23—4—1—1—4	



Secondly, An Elliptical Tun whose longest diameter is 5 gallons 8 primes and shortest diameter 5 gallons, 3 pri. (taken at each base by the circle square) and depth 49 inches, I demand its solidity in Ale barrels.

The Rule.

Multiply the longest Diameter by the shortest, &c.

gal-

	gall. ′	
	5—8	
	5—3	
	28—4	
	1—5	
On each wet inch — gallons	29—9	divided by the ratio's of 32, *viz.* 8 & 4.
	3—8—7—6	
barrels.	0-11—1—10—6	
	6-06—1—01—6	multip. by the ratio's of 49, *viz.* 7 and 7.
barrels in all	45-06—7—10—6	

Secondly, *Of Tuns that differ in their Dimensions at top and bottom.*

HEre I ſhall give you a new, and moſt exact, eaſie and ſpeedy way, for the finding the whole ſolid contents of all ſuch Tuns, and alſo to inch them. And for your better underſtanding thereof, you muſt know, that all ſuch Tuns may be cut into divers forms, as ſolid long ſquares, parallelopepidons, priſms, pyramids, *&c.* and ſo caſt up ſeverally; as when there is but one difference the Tun may be cut into a long ſolid ſquare, or parallelopepidon or one priſm, but if there be two differences then it may be cut into a parallelopepidon, two priſmes, and one pyramid; and this may be done not only when the ſides are flat, but when they are circular or Elliptical, (the ſlant ſides running ſtrait without ſuch curviture or bulging, as is in the ſpheroid, *&c.*) for working by the circle ſquare (which is the meaſure

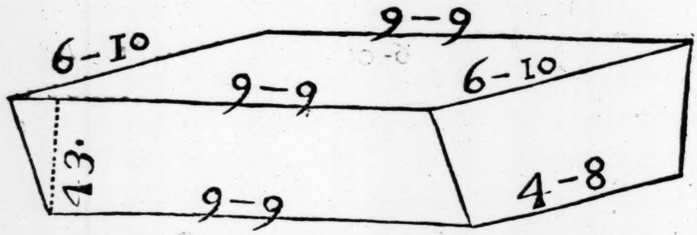
I work

I work all circular and Elliptical Tuns by) it reduces all circular or elliptical forms into ſquares, or parallelograms, as will clearly appear by the Examples hereafter given.

Firſt, I ſhall give you Examples of ſuch Tuns (differing in their dimenſions at top and bottom) whoſe ſides are flat, *viz.* bounded with right lines.

1. *Example.*

Of the Fruſtum of a Priſm.



A Tun 9 gallons 9 primes ſuperficial long at top and bottom, 6 gallons 10 primes broad at top, 4 gallons 8 primes broad at bottom, and 43 inches deep, how many gallons, and how many Beer barrels may this Tun contain?

Note, *For all flat ſided Tuns I work by the ſuperficial or ſquare Gallon.*

Now here obſerve, here is but one difference, and that is in the breadth or thickneſs, therefore you may cut this Tun into a parallelopepidon, and one priſm, and ſo caſt them ſeverally, but there being no difference in the length, you may (as a nearer way for this Tun and ſuch like) take an Arithmetical mean between the thickneſs at top, and the thickneſs at bottom, and multiply that mean by the length, and the product by the depth in inches.

The

The Operation.

gall. ′
6 — 10
4 — 8

11 — 6

5 — 9 Arithmetical mean.
9 — 9′ the length.

51 — 9
2 — 10 — 6
1 — 5 — 3

56 — 00 — 9 product.

336 — 04 — 6

2354 — 07 — 6
56 — 00 — 9

gallons — 2410 — 08 — 3

multiplied by the ratio's of 43.

401 — 09 — 4 — 6

barrels — 66 — 11 — 6 — 9

div. by the ratio's of 36

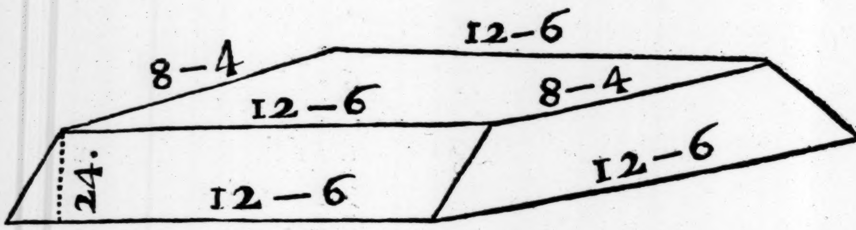
The whole Tuns capacity in —

I shall hereafter give a general Rule for the inching all such differing Tuns.

2. Exam-

2. *Example.*

Of the Frustum of a Prism.



There is a Tun whose base above is a Rectangular Parallelogram, *viz.* one side 12 gall. 6 primes and the other side 8 gall. 4 primes, and that below a perfect square; whose side is 12 gall. 6 primes, how many Ale gallons may the whole Tun contain when its depth is 24 inches?

This again may be cut into a Parallelopepidon, and a prism, the difference being but one, and that in thickness, but an Arithmetical mean being better I work it so. And note, if a difference had been in the length and none in the thickness, an Arithmetical mean had done the same. Note again, where an Arithmetical mean resolves the Problem, a Geometrical mean will be erroneous, so where a Geometrical mean resolves the Problem an Arithmetical mean will be erroneous.

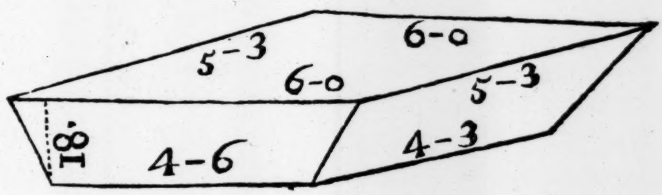
Opera-

Operation.

Gallons ,
12 — 6
8 — 4
———————
20 — 10
———————
10 — 5 Arithmetical mean
12 — 6 the length
———————
125 — 0
5 — 2 — 6
———————
130 — 2 — 6 product
6
———————
781 — 3 — 0
4
———————
gallons 3125 — 0 — 0
———————

multiplyed by the ratio's of 24 for the Tuns whole capacity.

3. *Example.*

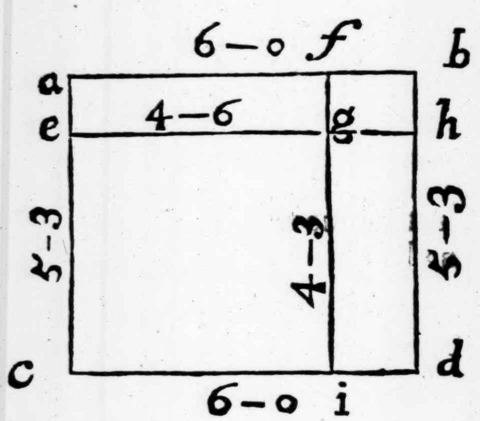


A Tun in the form of a Prismoid whose dimensions are as followeth, *viz.*

above— { gall. , / 6 —— 0 / 5 —— 3 } below — { gall. , / 4 —— 6 / 4 —— 3 } depth 18 inches } What may be the Tuns solidity in gallons?

Here

Here note, The differences are two, one in the length the other in the thickneſs, and therefore you may cut this Tun into a Parallelopepidon, 2 priſmes, and one pyramid, as you may ſee in the Figure, and ſo meaſure them, and caſt them up ſeverally according to Rules for ſuch forms before given, which may thus be performed, *viz.*



The Rule.

1. Find the Area of the leſſer baſe, and reſerve it.

2. Subtract the longeſt ſide of the leſſer baſe from the longeſt ſide of the greater baſe, and ſo the ſhorteſt ſide of the leſſer baſe, from the ſhorteſt ſide of the greateſt baſe.

3. Multiply the longeſt ſide of the leſſer baſe by half the difference of the ſhorteſt ſides, and the ſhorteſt ſide of the leſſer baſe by half the difference of the longeſt ſides, and add thoſe two products with the reſerved Area, and multiply the ſum by the inches of the depth, which product will be the ſolid content of the parallelopepidon and two priſms.

4. Multiply one difference by the other, and that product by one third of the depth in inches, and that product will be the ſolid content of the pyramid, which added with the former, will be the whole ſolid content of the Tun.

Demonſtration.

Let *a b c d* repreſent the baſe above, whoſe ſides are 6 and 5—3, and *e g c i* the baſe below whoſe ſides are

4—6

4—6 and 4—3, now if *a b c d* be cut in *f i* with a plane parallel to *a c* or *b d*, and if *a b c d* be cut again in *e h* with a plane parallel to *a b* or *c d*, it will plainly appear that the Tun will ſo be cut into the parallelopepidon *e g c i*, and two priſmes, *viz.* *a e f g* and *g h i d*, and the pyramid *f g b h*.

Operation.

4 — 6 longeſt ſide below
4 — 3 ſhorteſt ſide below

18 — 0
1 — 1 — 6

Area of 19 — 1 — 6 the leſſer baſe.

6 — 0 longeſt ſide above.
4 — 6 longeſt ſide below.

1 — 6 difference

5 — 3 ſhorteſt ſide above
4 — 3 ſhorteſt ſide below

1 — 0 difference

4 — 6 longeſt ſide below
0 — 6 ſemi-difference of the ſhorteſt ſides.

2 — 3 product

4 — 3 ſhorteſt ſide below
0 — 9 ſemi-differ.of the longeſt (ſides.

2 — 1 — 6
1 — 0 — 9

Areas of the two priſmes { 3 — 2 — 3
2 — 3 — 0
Area of 19 — 1 — 6 the leſſer baſe.

multiplyed by the ratio's of 18 { 24 — 6 — 9
6
147 — 4 — 6
3

442 — 01 — 6

1 — 6 } difference
1 — 0

1 — 6
6 the $\frac{1}{3}$ of the depth

9 — 0 cont.of the pyram.

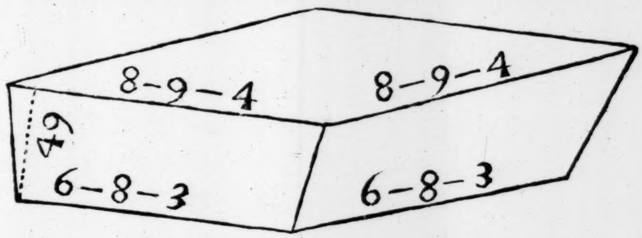
442 — 01 — 6
Solid cont.of the pyramid 9 — 0 — 0
content of the Tun 451 — 1 — 6

I think it no Hereſie in Geometry, to call that a pyramid, which terminates in a ſharp point, whether the baſe be a Circle, a Triangle, a poligon, a parallelogram, or a ſquare, only with theſe diſtinctions, *viz.* a round, a triangular, a poligonial, a parallelogram, and a ſquare Pyramid.

Note, The common way to perform this is to find the Areas of the two baſes, then to multiply one Area by the other, and to extract the ſquare root of the product, and add the two Area's and the ſquare root together, and multiply the ſumm by one third part of the depth.

3. *Exam-*

3. Example. Of the Frustum of a Pyramid.



A Tun in the form of the Frustum of a pyramid, the side of the greater end is 8 gall. 9 primes 4 seconds, and of the lesser 6 gall. 8 primes 3 seconds, the depth 49 inches, how many gallons and Beer barrels may this Tun contain.

6—8—3 } sides of lesser base
6—8—3 }

40—1—6
3—4—1—6
1—1—4—6
0—1—8—0—9

44—8—8—0—9 Area of the square

8—9—4 side above
6—8—3 side below

2—1—1 difference.

1—0—6—6 semi-difference.
6—8—3—0 side of lesser base.

6—3—3—0
6—3—3
2—1—1
0—3—1—7—6

6—11·—10·—5—7·—6 Area of one Prism.
6—11·—10·—5—7·—6 Area of the other prism.
44—8—8—0—9—0 Area of the lesser base.

O 3 58—8

Note, *Here are two differences though one and the same.*

```
58—8-05—·—··—··
         6
352—2-06               Multplied by the ra-
         8             tio's of the depth.—
2817—8—0
  58—8—5
2876—4—5 product

2—1—1  } difference.
2—1—1  }

4—2—2
  2—1—1
    2—1—1

4—4—5—2—1
        16—4 the 1/3 of depth.

17—5—8—8—4

69—10—10—9—4
 1—5—5—8—8—4

71—4—4—6—0—4 content of the pyramid.

2876—4—5
  71—4—4—6—0—4 solid cont. of pyramid
2947—8—9—6—0—4 Tuns whole content in gallons
 491—3—5—7—0—0—8
  81-10—6-11—2—0—1—4 in barrels
```

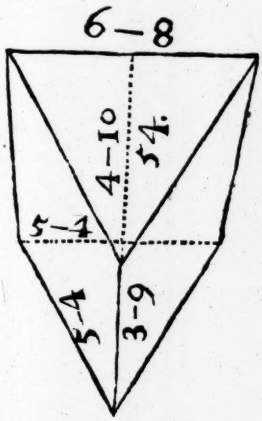
Ano-

Another way for a Pyramidall Tun.

To the Area's of each baſe, add the product of the greater ſide, multiplyed by the leſſer ſide, and multiply the ſum by the $\frac{1}{3}$ part of the depth in inches.

4. *Example.*

Of the Fruſtum of a Triangular Pyramid.



A Tun in the form of the Fruſtum of a Triangular Pyramid, the baſe of the Triangle above is 6 gall. 8 primes of the ſuperficial gallon, and the perpendicular 4 gall. 10 primes, and the baſe of the Triangle below is 5 gallons 4 primes, and the perpendicular 3 gall. 9 primes, how many Ale gallons may this Tun contain when its depth is 48 inches?

Now the baſe of a Triangle multiplyed by half the perpendicular, reduceth it into an oblong, or long ſquare, therefore the ſame Rule holds ſtill, for ſubtracting the baſe at the bottom from the baſe at the top, half the perpendicular at the bottom from half the perpendicular at the top, you will have two differences, and ſo the Tun will be cut into a parallelopepidon, two priſms, and a pyramid, and therefore muſt be wrought as a 4 ſided Priſmoid.

Operation.

gall. ,

below— { 1 — 10 — 6 the half of the perpendicular.
5 — 04 — 0 baſe

9 — 04 — 6
0 — 07 — 6

10 — 00 — 0 Area of the parallelogram.

Baſes { 6 — 8
5 — 4

1 — 4 difference

0 — 8 ſemi-difference

ſhorteſt ſide by half { 1 — 10 — 6
the differ. of longeſt { 0 — 8 — 0

0 — 11 — 3
3 — 9

product 1 — 03 — 0 Area of one priſm.

half perpendiculars { 2 — 05
1 — 10 — 6

difference—0 — 06 — 6

ſemi-difference 0 — 03 — 3

0 — 3

0 —— 3 —— 3 } longest side by half the difference of
5 —— 4 —— 0 } the shortest.

1 —— 4 —— 3
1 —— 1 —— 0

1 —— 5 —— 4 —— 0 product Area of the other prism.

0 —— 6 —— 6 —— 0 } one difference by the other.
1 —— 4 —— 0 —— 0 }

0 —— 6 —— 6
0 —— 2 —— 2

0 —— 8 —— 08 —— 0 product
16 the third of the depth.

2 —— 10 —— 8 —— 0

11 —— 6 —— 8 —— 0 content of the pyramid.

10 —— 00 —— 0 Area of the Parallelogram.
Areas of the prisms { 1 —— 03 —— 0
1 —— 05 —— 4

Total —12 — 08 —— 4
48

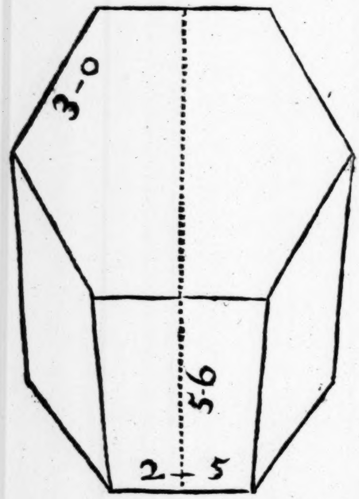
101 —— 06 —— 08

609 —— 04 —— 00
11 —— 06 —— 08 content of the pyramid.

620 —— 10 —— 08 content of the Tun.

5. *Exam-*

5. *Example.*
Of the Frustum of a Poligonial Pyramid.



A Tun in the form of the frustum of a Poligonial pyramid whose sides are 6, each side above being 3 gallons, each side below 2 gallons 5′ and depth 56 inches, how many gallons may this Tuns capacity be?

	gall.	′	″
the greatest perp. is	2	7	9
the least perpend. is	2	1	6

Note, half the perimeter (as you have been taught) multiplyed by the whole perpendicular, reduceth the poligon into a long square or oblong, so that here again will be two differences, the one between the 2 perpendiculars, the other between the two semi-perimeters, whereby the Tun will be cut as the former, therefore the same Rule will perform, not only this, but all other frustums of polygonial pyramids whatsoever, or how many soever their sides may be.

Operation.

6 sides each
3 gallons above

18 perimeter

9 semi-perimeter.

6 sides

6 sides each

2 — 5 below

14 — 6 perimeter

7 — 3 semi-perimeter.

1 — 9 difference

10 — 6 semi-difference

2 — 7 — 9 } perpendiculars

2 — 1 — 6 }

0 — 6 — 3 difference

0 — 3 — 1 — 6 semi-diff.

2 — 1 — 6 perpendicular below

0 — 10 — 6 semi-difference

1 — 0 — 9

8 — 6

1 — 0 — 9

product 1 — 10 — 3 — 9 Area of one prism.

7 — 3 — 0 semi-perimeter below

0 — 3 — 1 — 6 semi-difference

1 — 9 — 9 — 0

0 — 7 — 3

3 — 7 — 6

product 1 — 10 — 7 — 10 — 6 Area of the other pris.

1 — 9

1 — 9 — 0 } one difference by the
0 — 6 — 3 } other.

0 — 10 — 6
5 — 3

product 0 — 10 — 11 — 3
18 — 8 the third of 56.

5 — 5 — 7 — 6

16 — 4 — 10 — 6
3 — 07 — 9
3 — 07 — 9

content 17 — 0 — 02 — 0 of the pyramid.

2 — 1 — 6 perpendicular below
7 — 3 — 0 ſemi-perimeter.

14 — 10 — 6
6 — 4 — 6

15 — 4 — 10 — 6 Area of parallelogram.
1 — 10 — 3 — 9 — 0 } Areas of the priſms.
1 — 10 — 7 — 10 — 6 }

19 — 01 — 10 — 1 — 6 total
56 depth

153 — 02 — 9 — 0 — 0

1072 — 07 — 3 — 0 — 0
17 — 00 — 2 — 0 — 0 content of the pyramid.

1089 — 07 — 5 — 0 — 0 whole ſolid content of the Tun in Ale Gallons.

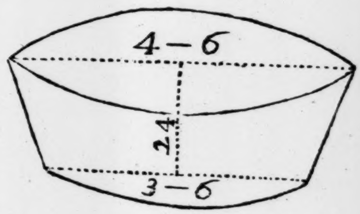
I come

I come now in the ſecond place to give you ſome Examples of Tuns, differing in their dimenſions at top and bottom, whoſe ſides are either circular or elliptical, the ſlant ſides running ſtrait without curviture or bulging.

1. *Example.*

Of the Fruſtum of a Cone.

A Tun in the form of the Fruſtum of a Cone, whoſe diameter at the top is 4 gallons 6 primes of the circle ſquare gallon, and its diameter at the bottom is 3 gallons 6 primes, and depth 24 inches, how many gallons may this Tun contain?



Note, *In taking your diameters of all round Tuns you muſt take them both ways acroſs, for if there be any difference, you muſt work as if it were Elliptical.*

Note again, A Cone may be called a round pyramid, and the ſame Rule I have given for a pyramidal Tun, muſt be obſerved for a Conical, and indeed the circle ſquare gallon (by which I work it) doth reduce it into the Fruſtum of a ſquare pyramid, for taking the diameters by the circle ſquare, the meaſures ſo taken thereon are the ſides of ſquares equal, which ſquared gives the Area or content, as if the diameters had been ſides of ſquares; therefore you may eaſily perceive that according to this method, in every fruſtum of a Cone, as well as in the fruſtum of a pyramid, there will be two differences, though both one and the ſame, (as I noted in the example of a pyramidal Tun,) but here note the leſſer diameter being to be multiplyed twice, by the ſemi-difference, (as is done in that of the pyramidal Tun, by doubling the product of once multiplying it) you

may

may multiply it onely once by the whole difference, and then proceed as you did for the pyramidal Tun, it being reduced by the circle ſquare into a long ſquare, two priſms, and one pyramid.

Operation.

3—6 } leſſer	4—6
3—6 } diamet.	3—6
10—6	1—0 difference
1—9	3—6 multiplyed
Area of 12—3 the ſquare.	3—6 Area's of 2 priſmes
Area of 3—6 the 2 priſms.	
their ſum 15—9 } mult. by the rati-	1—0 to mult. 1 by 1 is but 1
24 } o's of 24.	8—0 the $\frac{1}{3}$ of 24 the depth.
94—6	8—0 cont. of the pyramid.
378—0	
8—0 ſolid content of the pyramid.	
gall. 386—0 the Tuns ſolid capacity.	

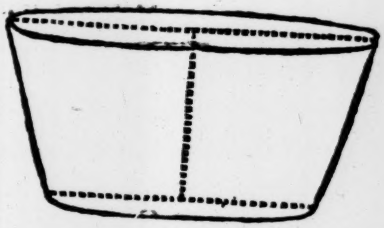
Another way for a Conical Tun.

To the Area's of the two baſes, add the product of one diameter multiplyed by the other, and their ſum multiply by one third of the depth in inches.

2. *Exam-*

2. *Example.*

Of the Frustum of an Elliptical Cone.



A Tun in the form of the Frustum of an Elliptical Cone, whereof the Axe or greater diameter at the top is 2 gallons 6 primes(of the circle square) its lesser diameter at the top is 2 gallons 1 prime: its greater diameter at bottom is 2 gallons 4 primes and its lesser diameter at bottom is 1 gallon, 10 primes, 2 seconds, and the depth 30 inches, how many Ale gallons is this Tuns capacity?

Note, here are two differences, one between the greater diameters and another between the lesser, so you must observe the same Rule before given, for the diameters being taken by the circle square, the Tun is reduced into a parallelopepidon, two prisms, and one pyramid.

Operation.

below— { 1 — 10 — 2 lesser diameter
{ 2 — 4 — 0 greater diameter.

3 — 8 — 4
7 — 4 — 8

Area of 4 — 3 — 8 — 8 parallelogram.

2 — 6

2 — 6 } greater diameters.
2 — 4 }

———————

0 — 2 difference

———————

0 — 1 ſemi-difference

———————

2 — 1 — 0 } leſſer diameters.
1 — 10 — 2 }

———————

0 — 2 — 10 difference

———————

0 — 1 — 5 ſemi-difference.

2 — 4 greater diameter below
by 0 — 1 — 5 ſemi-differ. of leſſer diam.

———————

2 — 4
0 — 9 — 4
0 — 2 — 4

———————

product 3 — 3 — 8 Area of one priſm.

———————

1 — 10 — 2 leſſer diameter below by
0 — 01 — 0 ſemi-differ.of great.diamet.

———————

product 1 — 10 — 2 Area of the other priſm

———————

2 — 10 leſſer diameter difference by
0 — 2 greater diameter difference.

———————

0 — 05 — 8 product
10 the third of the depth.

———————

4 — 8 — 8 content of the pyramid.

———————

Area

Area of 4— 3— 8—8 parallelogram.
Areas of { 3— 3—8 } two priſms.
1—10—2

4— 8—10—6 their ſum
30 depth.

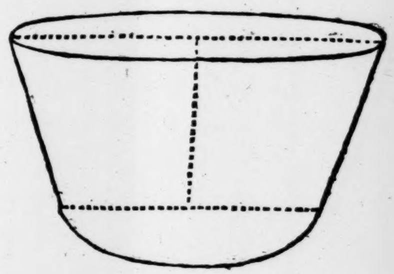
Multiplyed by the ratio's of 30.
28— 5— 3—0

144— 2— 3—0
Cont. of the 4— 8— 8—0 pyramid.

gallons in all 148—10—11—0

3. *Example.*

Of a Circulo-Elliptical Solid called a Cylindroid.



A Tun Circular below, and Elliptical above, the diameter of the circular baſe, is 2 gall. 4 primes, 7 ſeconds, the Axe or greater diameter of the Elliptical baſe is 2 gallons 8 primes, 2 ſeconds, and the leſſer diameter 2 gallons 7 primes, the depth 30 inches, how many Ale gallons is this Tuns capacity?

Note, Here are two differences, therefore the work will be the ſame with the former, only obſerve; that the diameter of the circle muſt be taken out of both the diameters of the Ellipſis, then you will have one and the ſame diameter to be multiplyed by both the ſemi-differences, which done proceed as before.

Operation.

2—4—7 circles diameter.
2—4—7

4—9—2
9—6—4
1—2—3—6
2—4—7

5—8—1—0—1 Area of the ſquare.

2—8—2 greater diameter above.
2—4—7 circles diameter.

0—3—7 difference.

2—7—0 leſſer diameter above.
2—4—7 circles diameter.

0—2—5 difference.

2—4—7 circles diameter.
0—3—7 difference.

1—4—8—1
0—7—1—9

0—8—6—5—1
0—5—9—0—11

½—1—2—3—6—0

0—7—1—9—0 content of the priſms.

2—4—7 circles diameter
0—2—5 differ.

11—10—11
4—9—2

0—5—9—0—11

0—3

0——3——7 one difference by
0——2——5 the other.

0——0——7——2
1——2——4
3——7

0——0——8——7——11 product
10 a third of the depth.

0——7——2——7——2 cont. of the pyramid

5——8——1——0——1 Area of the square.
7——1——9——0 Area of the 2 prisms

6——3——2——9——1 their sum.
30 depth.

37——7——4——6——6

188——0——10——8——6
0——7——2——7——2 cont. of the pyramid

188——8——1——3——8 content of the Tun
(in gallons

A most Exact, Brief, and Easie way, for the Inching of all the aforesaid Tuns differing in their dimensions at Top and Bottom.

NOw I shall shew you how you may find the solid contents of any of the aforesaid Tuns, differing in their dimensions at top, and bottom, at every Inch throughout their depth or perpendicular, or at any part thereof. And that you may the better understand the reason of it, you must know that upon every Inch, or upon any part of the Perpendicular, there will be, (as there are upon the whole) two Prisms, and one Pyramid being cut with 2 Planes parallel to the sides, and the perpendicular will still proportion them to the whole; For if you divide each difference between the two opposite sides, at the top, and bottom by the perpendicular, the quote of the difference between the two longest sides, will be the thickness of the shortest prism, and one side of the pyramid, and the quote of the difference between the two shortest sides, will be the thickness of the longest prism, and the other side of the pyramid, for one inch of the depth or perpendicular; Wherefore,

First, If you Multiply one quote by the other, the $\frac{1}{3}$ of the product or rectangle will be the content of the pyramid at one inch of the depth, or perpendicular, which multiplyed by the * Cube of the perpendicular, will be the solid content of the Tuns whole Pyramid. Se-

* Viz. *By the square of the perpendicular, (both differences being divided by it, and the rectangle made by the two quotes,) and that square by the perpendicular for the depth.*

Secondly, If you multiply the ſide of the longeſt priſm, *viz.* the longeſt ſide of the leſſer Baſe, by the quote of the difference of the ſhorteſt ſides, and the ſide of the ſhorteſt priſm, *viz.* the ſhorteſt ſide of the leſſer Baſe by the quote of the difference of the longeſt ſides, the ſemy ſum of the two products, will be the content of the two priſms at one inch of the depth or perpendicular, which multiplyed by the * ſquare of the perpendicular, will be the ſolid content of the whole Tuns two priſms.

* Viz. *By the perpendicular, for multiplying each ſide by one quote, and that product by the perpendicular again for the depth.*

Thirdly, If you multiply the Area of the leſſer Baſe by the perpendicular, and add the product with the contents of the Tuns pyramid, and two priſms, you will have the whole content of the Tun.

And this may very eaſily be performed, if you obſerve this following order; *Viz.*

Multiply the content of the pyramid at one inch of the depth by the perpendicular, and to the product add the content of the two priſms, and multiply the ſum by the perpendicular, and to that product add the Area of the leſſer Baſe, and multiply the ſum by the perpendicular, that product will be the whole ſolid content of the Tun, for ſo will the pyramid be multiplyed by the Cube, and the two Priſms by the ſquare of the perpendicular. And the Area of the leſſer Baſe by the perpendicular.

And by the ſame method, you may find the ſolid contents at every inch throughout the depth of the Tun, or perpendicular, or at any part thereof.

For Examples, I will take two Priſmoidal Tuns, one differing much, the other but little in their dimenſions at top and bottom, and this way find their whole ſolid contents, and then their ſolid contents at each inch, and make

a Table of each; and for brevity's ſake, ſhall make neither of them above 9 inches deep, which will be ſufficient for your Inſtruction, the ſame Rule performing all the reſt.

1. *Example.*

A Tun in the form of a priſmoid, whoſe dimenſions are as followeth. *viz.*

above { gall. 12, 8 } below { gall. 6, 4 } depth 9 inches } How many gallons may this Tun contain, and how many at each inch.

Operation.

9) 4 — 0 (0 — 5 — 4 } Quotes.
9) 6 — 0 (0 — 8 — 0 }

0 — 1 — 9 — 4
0 — 1 — 9 — 4

$\frac{1}{3}$ — 0 — 3 — 6 — 8 Rectangle.

cont. of the pyramid — 0 — 1 — 2 — 2 — 8 at 1 inch.

12 } Two greater oppoſite ſides.
6 }

6 difference

8 } Two leſſer oppoſite ſides.
4 }

4 difference.

6 The

6 The longest side of lesser base.
0 — 5 — 4 quote of the difference of the shortest.

2 — 8 — 0 product

4 The shortest side of lesser base.
0 — 8 Quote of the difference of the longest.

2 — 8 product
2 — 8

½ — 5 — 4

2 — 8 content of the two prisms at one inch.

6 } sides of the lesser base.
4 }

24 Area

0—1—2—2—8 cont. of the pyramid at one inch deep.
9 perpendicular.

0—10—8—0—0 product
2—8—0—0—0 cont. of the 2 prisms at 1 inch deep.

3—6—8—0—0 Sum
9 perpendicular

32—0—0—0—0 product
24—0—0—0—0 Area of the lesser base.

59—0—0—0—0 sum
9 perpendicular

504—0—0—0—0 Answer whole content of the Tun.

Now suppose 5 inches of the perpendicular were wet, how many gallons would there then be in it?

0 — 1 — 2 — 2 — 8 pyramid
5

0 — 5 — 11 — 01 — 4
2 — 8 — 0 — 0 — 0 prisms

3 — 1 — 11 — 1 — 4
5

15 — 9 — 07 — 6 — 8
24 — 0 — 0 — 0 — 0 Area

39 — 9 — 7 — 6 — 8
5

199 — 0 — 1 — 9 — 4 answer

Now to find what it contains at every inch.

0 — 1 — 2 — 2 — 8 pyramid
2 — 8 — 0 — 0 — 0 prisms
24 — 0 — 0 — 0 — 0 Area

Note, *as I will neither multiply nor divide, so will it neither square nor Cube.*

26 — 9 — 2 — 2 — 8 content at the first inch.

0 — 1 — 2 — 2 — 8 pyramid
2 inches

0 — 2 — 4 — 5 — 4 product
2 — 8 — 0 — 0 — 0 prisms

2 — 10 — 4 — 5 — 4 sum
2 inches

5 — 8 — 8 — 10 — 8 product
24 — 0 — 0 — 0 — 0 Area

29 — 8 — 8 — 10 — 8 sum
2 inches

59 — 5 — 5 — 09 — 4 content at the second inch.

0 — 1

0 — 1 — 2 — 2 — 8 pyramid
3 inches

---

0 — 3 — 6 — 8 — 0
2 — 8 — 0 — 0 — 0 prisms

---

2 — 11 — 6 — 8 — 0
3 inches

---

8 — 10 — 8 — 0 — 0
24 — 0 — 0 — 0 — 0 Area

---

32 — 10 — 8 — 0 — 0
3 inches

---

98 — 8 — 0 — 0 — 0 content at the third inch.

---

0 — 1 — 2 — 2 — 8 pyramid
4 inches.

---

0 — 4 — 8 — 10 — 8
2 — 8 — 0 — 0 — 0 prisms

---

3 — 0 — 8 — 10 — 8
4 inches

---

12 — 2 — 11 — 6 — 8
24 — 0 — 00 — 0 — 0 Area

---

36 — 2 — 11 — 6 — 8
4 inches

---

144 — 11 — 10 — 2 — 8 content at the fourth inch.

The

The Table of this Tun.

inch.	gall.	′	″	‴	⁗
9	504	00	00	00	00
8	413	02	09	09	04
7	340	08	06	02	08
6	261	04	00	00	00
5	199	00	01	09	04
4	144	11	10	02	08
3	98	08	00	00	00
2	59	05	05	09	04
1	26	09	02	02	08

If you go on in the ſame manner throughout the Tun you will find it at every inch as in the Table above. And if you divide the contents at each inch by the ratio's of 36 the gallons will be reduced into Beer barrels and the Table will ſtand thus.

inch.	barr.	′	″	‴	⁗	′′′′′	′′′′′′
9	14	00	00	00	00	00	00
8	11	05	08	11	03	01	04
7	09	05	06	10	00	10	08
6	07	07	01	04	00	00	00
5	05	06	04	00	07	01	04
4	04	00	03	11	04	10	08
3	02	08	10	08	00	00	00
2	01	07	09	09	11	01	04
1	00	08	11	00	08	10	08

Now

Now if this Tun were ſtanding on its great baſe, and you were to inch it upward, you muſt obſerve this following order, *viz.*

Find the difference of the oppoſite ſides above, and below, as you did before, and divide both differences by the perpendicular. Then multiply one quote by the other, and take the third of the product for the content of the pyramid, at one inch of the depth, as you did before, then multiply the longeſt ſide of the greater baſe, by the quote of the difference of the ſhorteſt ſides, and the ſhorteſt ſide of the greater baſe by the quote of the difference of the longeſt ſides, and add the two products together, and take the ſemi-ſum for the content of the two priſms, at one inch of the depth, then find the Area of the greater baſe, and ſo proceed with the pyramid, priſms and Area as you did before; onely as you then ſucceſſively added the priſms and the Area, here you muſt ſubtract the product of the pyramid multiplyed by the perpendicular from the priſms, and multiply the remain by the perpendicular, and ſubtract that prodnct from the Area, and multiply the remain by the perpendicular, and that product will be the whole content of the Tun. And after the ſame manner you muſt work for 1 inch, 2 inches, 3, *&c.* or for any part of the Tun.

And for an Example I will ſuppoſe the ſame Tun to ſtand on its greater baſe.

$\left.\begin{matrix}6\\4\end{matrix}\right\}$ differences.

---

9) 6—0 (0—8—0 } quote of the dif of longeſt ſides
9) 4—0 (0—5—4 } quote of the dif. of ſhorteſt ſides.

0—8—0

0—8—0
0—5—4

$\frac{1}{3}$—3—6—8

1—2—2—8 pyramid

12 longest side of the greater base.
0—5—4 quote of the difference of the shortest sides.

5—4—0 product.

8—0 shortest side of the greater base.
0—8 quote of the difference of the longest sides.

5—4 product
5—4

$\frac{1}{2}$—10—8

5—4 prisms

12
8

96 Area of greater base.

Now with these Three, *viz.* { 0—1—2—2—8 pyramid. 5—4—0—0—0 prisms. 96—0—0—0—0 Area. }
I proceed.

0—1

0—1—2—2—8 pyramid
9 perpendicular

0-10-8—0—0 product ſubtracted from the
5—4—0—0—0 priſms

4—5—4—0—0 remain
9 perpendicular

40—0—0—0—0 product ſubtracted from the
96—0—0—0—0 Area

56—0—0—0—0 Remain
9 perpendicular.

504—0—0—0—0 whole content of the Tun.

Now ſuppoſe 6 inches of the perpendicular were wet how many gallons would there be then in the Tun, as ſtanding on its greater baſe ?

0—1—2—2—8 pyramid
6 inches

0—7—1—4—0 product ſubtracted from the
5—4—0—0—0 priſms

4—8-10-8—0 Remain
6 inches

28—5—4—0—0 product ſubtracted from the
96—0—0—0—0 Area

67—6—8—0—0 Remain
6

405—4—0—0—0 anſwer content at 6 inches.

Now

Now to find the contents at every inch.

0—1—2—2—8 pyramid ſubtracted from the
5—4—0—0—0 priſms.

5—2—9—9—4 remain ſubtracted from the
96—0—0—0—0 Area

90—9—2—2—8 remain, content at 1 inch deep.

0—1—2—2—8 pyramid
2 inches

0—2—4—5—4 product ſubtracted from the
5—4—0—0—0 priſms

5—1—7—6—8 remain
2 inches

10—3—3—1—4 product ſubtracted from the
96—0—0—0—0 Area

85—8—8—10—8 Remain
2 inches

171—5—5—9—4 product, content at the ſecond inch.

And if you go on after the ſame manner throughout the Tun, you will find the Tun to contain at every inch, as in the following Table.

Which

Which may be reduced into barrels as formerly whether of Beer or Ale.

inch.	gall.	′	″	‴	⁗
9	504	00	00	00	00
8	477	02	09	09	04
7	444	06	06	02	08
6	405	04	00	00	00
5	359	00	02	09	04
4	305	03	10	03	08
3	242	08	00	00	00
2	171	05	05	09	04
1	90	09	02	02	08

But if you would have your anſwers in Beer or Ale barrels at the firſt working, obſerve this method, *viz.* for Beer take the third of the $\frac{1}{12}$ of each of the 3 numbers you are to work by, *viz.* the pyramid, priſms, and Area ; and for Ale take the fourth of the $\frac{1}{8}$ of each of thoſe numbers, and work by the numbers ſo taken as before. So the numbers for this Tun as ſtanding on its leſſer Baſe, for

Beer will be { 0-0- 0-4-8-10-8 Pyramid
0-0-10-8-0-00-0 Priſms
0-8-00-0-0-00-0 Area }

Ale will be { 0-0-0-5-4 Pyramid
0-1-0-0-0 Priſms
0-9-0-0-0 Area }

But as ſtanding on its greater baſe ; the numbers for

Beer will be { 0-0-0-4-8-10-8 Pyramid
0-1-9-4-0- 0-0 Priſms
2-8-0-0-0- 0-0 Area }

Ale

Ale will be { 0-0-0-5-4 Pyramid
0-2-0-0-0 Prisms
3-0-0-0-0 Area }

2. *Example*

A Tun in the form of a Prismoid whose dimensions are,

Above { gall. ,
6 —— 8
7 —— 10 }

Below { 5 —— 10
6 —— 9 }

depth 9 inches

How many gallons may it contain, and how many at each inch?

The Operation.

6 —— 8 } opposite sides.
5 —— 10 }

0 —— 10 difference

7 —— 10 } opposite sides
6 —— 9 }

1 —— 01 difference

9) 0 —— 10

9) 0—10(0—1—1—4 quote of the differ. shortest sides

1—0

3—0

9) 1—1 (0—1—5—4 quote of differ. of longest sides

4—0

3—0

0—1—5—4
0—1—1—4

0—0—1—5—4
1—5—4
5—9—4

⅓—0—0—1—7—3—1—4

pyramid 0—0—0—6—5—0—5—4

0-1-1-4
6-9-0-0

0-6-8-0
0-6-8
3-4

0-7-6-0
0-8-4-1-4

⅓—1-3-10-1-4

0-7-11-0-8 prisms

0-1-5-4
5-10-0-0

7-1-8
0-8-8
5-9-4

0-8-4-1-4

5 — 10

6 — 9

---

35 — 0

2 — 11

1 — 5 — 6

---

39 — 4 — 6 Area

---

0 — 0 — 0 — 6 — 5 — 0 — 5 — 4 pyramid

9

---

0 — 0 — 4 — 9 — 9 — 4 — 0 — 0

0 — 7 — 11 — 0 — 8 — 0 — 0 — 0

---

0 — 8 — 3 — 10 — 5 — 4 — 0 — 0

9

---

6 — 2 — 10 — 10 — 0 — 0 — 0 — 0

39 — 4 — 6 — 0 — 0 — 0 — 0 — 0

---

45 — 7 — 4 — 10 — 0 — 0 — 0 — 0

9

---

410 — 6 — 7 — 6 — 0 — 0 — 0 — 0 whole content of the Tun.

Now if you inch it as you have been taught, you will find the Tun to contain at every inch as in the Table, *viz.*

inch.

inch.	gall.	′	″	‴	⁗	′′′′′	′′′′′′	′′′′′′′
9	410	06	07	06	00	00	00	00
8	359	01	09	05	06	11	06	08
7	309	02	11	03	02	08	05	04
6	260	09	09	06	08	00	00	00
5	213	09	11	02	01	07	06	08
4	168	03	07	01	06	04	05	04
3	124	01	01	11	11	04	00	00
2	81	04	00	06	00	01	06	08
1	40	00	05	07	01	00	05	04

And that you may ſee that all the former Examples concerning Tuns, differing in their dimenſions at top and bottom, may be meaſured and inched the ſame way, I will here again take the Conical Tun before given, whoſe diameter at top was 4 gallons 6 primes of the circle ſquare gallon, and its diameter at bottom 3 gallons 6 primes and the perpendicular 24 inches, and its whole content found to be 386 gallons.

First to find its whole content.

4 — 6
3 — 6

24) 1 — 0 (0 — 0 — 6 quote
0 — 0 — 6

0 — 0 — 0 — 3 — 0

pyramid 0 — 0 — 0 — 1 the third

 0 — 0

0 — 0 — 6
3 — 6 — 0

0 — 1 — 6
0 — 0 — 3

prisms 0 — 1 — 9

3 — 6
3 — 6

10 — 6
1 — 9

Area 12 — 3

0 — 0 — 0 — 1 pyramid
24

0 — 0 — 2 — 0
0 — 1 — 9 — 0 prisms.

0 — 1 — 11 — 0
24

0 — 11 — 6 — 0

3 — 10 — 0 — 0

by the ratio's of 24, *viz.* 6 and 4.

12 — 03 — 0 — 0 Area

16 — 01 — 0 — 0
24

96 — 6 — 0 — 0

386 — 0 — 0 — 0 whole content being the same again.

Now suppose 15 inches were wet, how many gallons would be remaining.

0 — 0

0 —— 0 —— 0 —— 1
15

————————————

0 —— 0 —— 1 —— 3
0 —— 1 —— 9 —— 0

————————————

0 —— 1 —— 10 —— 3
15

————————————

0 —— 9 —— 3 —— 3

————————————

2 —— 3 —— 9 —— 9
12 —— 3 —— 0 —— 0

————————————

14 —— 6 —— 9 —— 9
15
72 —— 10 —— 0 —— 9

————————————

218 —— 6 —— 2 —— 3 answer in gallons

————————————

Now to inch it.

0 —— 0 —— 0 —— 1 pyramid
0 —— 1 —— 9 —— 0 prisms
12 —— 3 —— 0 —— 0 area

————————————

12 —— 4 —— 9 —— 1 content at 1 inch.

————————————

0	0	0	1	
			2	
0	0	0	2	
0	1	9	0	
0	1	9	2	
			2	
0	3	6	4	
12	3	0	0	
12	6	6	4	
			2	
25	1	0	8	content at 2 inches.
0	0	0	1	
			3	
0	0	0	3	
0	1	9	0	
0	1	9	3	
			3	
0	5	3	9	
12	3	0	0	
12	8	3	9	
			3	
38	0	11	3	content at 3 inches.

0——0——0——1
4

0——0——0——4
0——1——9——0

0——1——9——4
4

0——7——1——4
12——3——9——0

12——10——1——4
4

51——4——5——4 content at 4 inches.

So if you go through all the perpendiculars, you will find the contents at every inch as in the Table.

inch.	gall.	′	″	‴
24	386	00	00	00
23	365	11	02	11
22	346	02	11	04
21	326	11	00	09
20	307	11	06	08
19	289	04	04	07
18	271	01	06	00
17	253	02	10	05
16	235	08	05	04
15	218	06	02	03
14	201	08	00	08
13	175	02	00	01
12	169	00	00	00
11	153	01	11	11
10	137	07	11	04
9	122	05	09	09
8	107	07	06	08
7	093	01	01	07
6	078	10	06	00
5	064	11	07	05
4	051	04	05	04
3	038	00	11	03
2	025	01	00	08
1	012	04	09	01

If you would have your anſwers in Beer Barrels, then will be the.

$$\left.\begin{array}{l}\text{pyramid}\\ \text{priſms}\\ \text{area}\end{array}\right\}\begin{array}{l}0\text{-}0\text{-}0\text{-}0\text{-}0\text{-}4\\ 0\text{-}0\text{-}0\text{-}7\\ 0\text{-}4\text{-}1\end{array}$$

And working by theſe numbers as you wrought for gallons you will find its whole content in Beer barrels to be 10 barr. 8 primes, 8 ſeconds.

And if you would have it in Ale barrels, then will be the

$$\left.\begin{array}{l}\text{pyramid}\\ \text{priſms}\\ \text{area}\end{array}\right\}\begin{array}{l}0\text{-}0\text{-}0\text{-}0\text{-}0\text{-}4\text{-}6\\ 0\text{-}0\text{-}0\text{-}7\text{-}10\text{-}6\\ 0\text{-}4\text{-}7\text{-}1\text{-}6\end{array}$$

And working by theſe numbers as before you will find the Tuns whole content in Ale barrels will be 12 barrels 0 primes 9 ſeconds.

And after the ſame manner it may for both be inched.

But

But if this Tun were ſtanding on its greater baſe then would be for gallons the

Pyramid	0 —	0 —	0 —	1
Priſms	0 —	2 —	3	
Area	20 —	3 —	0	

For Beer barrels.

Pyramid	0 —	0 —	0 —	0 —	0 —	4
Priſms	0 —	0 —	0 —	9		
Area	0 —	6 —	9			

For Ale Barrels.

Pyramid	0 —	0 —	0 —	0 —	0 —	4 —	6
Priſms	0 —	0 —	0 —	10 —	1 —	6	
Area	0 —	7 —	7 —	1 —	6		

And here note, if you were to inch a ſtanding Conical Cask, you muſt ſuppoſe it to be cut into two equal parts at the Bung, by a plane parallel to either of the diameters at the heads, and then inch the lower part as ſtanding on its leſſer baſe, and the higher part as ſtanding on its greater baſe. But to the content at every inch of the higher part, you muſt add the whole content of the lower part.

Now I ſhall ſhew you how to inch the two Fruſtums of a priſm before given, one of them ſtanding on its leſſer baſe, the other on its greater, whoſe dimenſions were as followeth, *viz.*

Of the Firſt.

gall. ,
9 — 9 long at top and bottom.
6 — 10 broad at top.
4 — 8 broad at bottom.
Perpendicular 43 inches.

Here

Here being but one difference, if you divide the difference by the perpendicular, and by half the quote multiply the longeſt ſide, or the longeſt ſide by the quote, and take half of the product (which is all one) you will have the content of the priſm, at one ineh, which multiplyed by the perpendicular, and to the product add the Area of the leſſer baſe, and multiply the ſum by the perpendicular, you will have the whole content of the Tun. So if you would know the content at any part of the depth, multiply the priſm by the part, and add the Area of the leſſer baſe to the product, and multiply the ſum by the part, and the product will be the content of the part.

And by this Rule you will find the priſm to be — 0—2—11—4—5—7
Area of the leſſer baſe — — 45—6— 0—0—0—0

And ſo by theſe two numbers you may inch it throughout, Thus

0 — 2 — 11 — 4 — 5 — 7 priſm
45 — 6 — 00 — 0 — 0 — 0 Area

45 — 8 — 11 — 4 — 5 — 7 content at 1 inch.

0 — 2 — 11 — 4 — 5 — 7
2

0 — 5 — 10 — 8 — 11 — 2
45 — 6 — 0 — 0 — 0 — 0

45 — 11 — 10 — 8 — 11 — 2
2

91 — 11 — 9 — 5 — 10 — 4 content at 2 inches.

0—2

0	2	11	4	5	7
					3
0	8	10	1	4	9
45	6	0	0	0	0
46	2	10	1	4	9
					3
138	8	6	4	2	3 content at 3 inches

So if you go through the whole perpendicular, after the same manner, you will find the content on the last inch will be 2410—8—3—0—3—7 (gall. ′ ″ ‴ ⁗ ⁗′) the same with the former only a very small and inconsiderate matter more, that being 2410—8—3 which difference was occasioned by the fraction on the Division, for dividing the difference by the perpendicular, the quote ran thus, *viz.* 0-0-7-3-0-10-0-6-8-4-4 and so it would have ran 4 *ad infinitum*, then multiplying the longest side by the quote, when I came to take the half of the product, I found it to be 0-2-11-4-5-6-11-8-7-9-1-6 then considering the quote (my multiplyer) being somewhat too little, and finding a 11 at some distance from unity, I made the 11 to be 12 and so carryed the 1 to the next place, where stood 6 and made that 7 and there terminated the number, and so made use of 0-2-11-4-5-7 instead of 0-2-11-4-5-6-11-8-7-9-1-6 which if well observed will save much trouble when such large Fractions do arise.

If you would answer it in Beer barrels take the third of the $\frac{1}{12}$ of the prism, and the same of the Area, and work by those numbers as you did before for gallons.

If in Ale barrels take the $\frac{1}{4}$ of the $\frac{1}{8}$ of the prism, and the same of the Area, and work as before.

Of

Of the Second.

Whose dimensions were, *viz.* 12 gallons 6 primes long above, and 8 gallons 4 primes broad, and the base below a perfect square, whose side was 12 gallons 6 primes, and its depth 24 inches.

Now here is but one difference as there was in the former, therefore the same Rule for the prism, and the Area that is there given, must also be here observed, onely with this difference, this Tun standing on its greater base, and that on its lesser, you must here find the Area of the greater base, and when you have multiplyed the prism by the whole or any part of the depth, you must subtract the product from the Area, and multiply the remain by the whole, or by that part of the depth you work for, and so you will find for the Tun, the

Prism --	to be	1 — 1 — 0 — 3	for gallons
Area --		156 -— 3	

With which numbers I thus proceed for the inching it, *viz.*

```
  1 — 1 — 0 — 3
156 — 3
———————————————
155 — 1 — 11 — 9 content at 1 inch.
———————————————

  1 — 1 — 0 — 3
               2
———————————————
  2 — 2 — 0 — 6
156 — 3
———————————————
154 — 0 — 11 — 6
               2
———————————————
308 — 1 — 11 — 0 content on 2 inches.
———————————————
```

```
  1 —— 1 —— 0 —— 3
                 3
—————————————————
  3 —— 3 —— 0 —— 9
156 —— 3
—————————————————
152 —— 11 —— 11 —— 3
                   3
—————————————————
458 —— 11 —— 9 —— 9 content on 3 inches.
—————————————————
```

So if after the ſame manner you go through the whole perpendicular, you will find the Tun to contain at the laſt inch 3125 gallons.

But if you would have your anſwers in barrels, then according to the former Rule, for

Beer, your {Priſm / Area} will be {0-4-4-1 / 4-4-1}

Ale, your {Priſm / Area} will be {0-0-4-10-7-1-6 / 4—10—7—1—6}

As for the inching the reſt of the Tuns differing in their dimenſions at top and bottom before mentioned, I conceive I have given you ſufficient inſtruction, and therefore I ſhall leave them to your own practice.

Onely a little to help you, I ſhall give you the numbers of each, whereby you are to work them, that you may try to find the numbers the better your ſelf.

And firſt for the Priſmoid, as ſtanding on its leſſer baſe, the numbers will be

For

For Gallons.

Pyramid 0 — 0 — 0 — 2 — 8
Prisms — 0 — 3 — 7 — 6
Area — 19 — 1 — 6

For Beer Barrels.

Pyramid 0 — 0 — 0 — 0 — 0 — 10 — 8
Prisms — 0 — 0 — 1 — 2 — 6
Area — 0 — 6 — 4 — 6

For Ale Barrels.

Pyramid 0 — 0 — 0 — 0 — 1
Prisms — 0 — 0 — 1 — 4 — 3 — 9
Area — 0 — 7 — 2 — 0 — 9

But if it were standing on its greater base, then the numbers would be.

For Gallons.

Pyramid 0 — 0 — 0 — 2 — 8
Prisms — 0 — 4 — 7 — 6 — 0
Area — 31 — 6 — 0 — 0 — 0

For Beer Barrels.

Pyramid 0 — 0 — 0 — 0 — 0 — 10 — 8
Prisms — 0 — 0 — 1 — 6 — 6
Area — 0 — 10 — 6

For Ale Barrels.

Pyramid 0 — 0 — 0 — 0 — 1
Prisms — 0 — 0 — 1 — 8 — 9 — 9
Area — 3 — 11 — 3

Se-

Secondly, for the Frustum of the square Pyramid, as standing on its lesser base, the numbers will be

For Gallons.

Pyramid 0 — 0 — 0 — 1 — 0 — 6 — 11 — 3
Prisms — 0 — 3 — 5 — 0 — 11 — 7
Area — 44 — 8 — 8 — 0 — 9

For Beer barrels.

Pyramid 0—0—0—0—0—4—2—3—9
Prisms—0—0—1—1—8—3—10—4
Area — 1—2—10—10—8—3

For Ale Barrels.

Pyramid 0—0—0—0—0—4—8—7—2—7—6
Prisms—0—0—1—3—4—10—4—1—6
Area —1—4—9—3—0—3—4—6

But if it were standing on its greater base, then the numbers would be

For Gallons.

Pyramid 0 — 0 — 0 — 1 — 0 — 6 — 11 — 3
Prisms—0 — 4 — 5 — 11 — 0 — 7
Area — 77 — 0 — 7 — 1 — 4.

For Beer barrels.

Pyramid 0—0—0—0—0—4—2—3—9
Prisms—0—0—1—5—11—8—2—4
Area—2—1—8—2—4—5—4

For

For Ale Barrels.

Pyramid 0—0—0—0—0—4—8—7—2—7—6
Prisms—0—0—1—8—2—7—8—7—6
Area -—2 -4 -- 10—8—8

Thirdly, For the Frustum of the Elliptical Cone as standing on its lesser base, the numbers will be

For Gallons.

Pyramid 1-0-0-0-0-3-7-6-2-10-4-8-8-3-10-1
Prisms—0-0-2-0-8-9-7-1-8-2-7
Area 4-3-8-8

For Beer Barrels.

Pyramid 0-0-0-0-1-2-6-0-11-5-6-10-9-3-4-4
Prisms —0-0-0-0-8-2-11-2-4-6-8-10-4
Area—0-1-5-2-10-8-0

For Ale Barrels.

Pyramid 0-0-0-0-0-1-4-3-10-0-10-9-3-1-1-10
Prisms—0-0-0-0-9-3-3-7-2-1-6-11-7
Area —0-1-7-4-9

But if it were standing on its greater base, then the numbers will be

For Gallons.

Pyramid 0-0-0-0-3-7-6-2-10-4-8-8-3-1-0-1
Prisms—0-0-1-8-11-11-11-11-2-9-6
Area — 5-2-6

For

For Beer Barrels.

Pyramid 0-0-0-0-0- 1- 2- 6- 0-11- 5-6-10-9-0-4-0-4
Prisms—0-0-0-0-6-11-11-11-11- 8-11-2
Area —-0-1-8-10

For Ale Barrels.

Pyramid 0-0-0-0-0-1-4-3-10-0-10-9-3-1-1-10-6
Prisms – 0-0-0-0-7-10-5-11-11-8-6-6
Area—-0-1-11-5-3

But here obſerve I have ran the fractions to a great extent to ſhew the exactneſs of this way of working, but you may abbreviate them very much if you pleaſe, thus, *viz.* For every 10 inches of the perpendicular add * a third to the figure in the fourth place of each quote, and caſt off all the reſt, and ſo proceed with them as if they were the true quotes, and you will find it will come near enough the truth. And ſo I have done for the next Tun.

* *If you find it too little you may add more by your own diſcretion.*

Fourthly, For the Circulo Elliptical ſolid, called a Cylindroid, as ſtanding on its leſſer baſe, the numbers will be

For Gallons.

Pyramid 0 — 0 — 0 — 0 — 0 — 5
Prisms — 0 — 0 — 2 — 10 — 1 — 10 — 6
Area — 5 — 8 — 1 — 0 — 1

For Beer Barrels.

Pyramid 0 — 0 — 0 — 0 — 0 — 0 — 1 — 8
Prisms—0 — 0 — 0 — 0 — 11 — 4 — 7 — 6
Area — 0 — 1 — 10 — 8 — 4 — 0 — 4

For Ale Barrels.

Pyramid 0-0-0-0-0-0-1-10-6
Prisms—0-0-0-1-0-9-8-5-3
Area-—0-2-1-6-4-6-4-6

Note, By adding but a third for every 10 inches of the perpendicular it makes it here 1 gallon too much, which I have done on purpose to let you see the reason of it, *viz.* That the less is subtracted the more will be the remain, therefore you may add somewhat more as your discretion may guide you, which by a little practice will grow familiar, *&c.*

As standing on his greater base, the numbers will be

For Gallons.

pyramid 0——0——0——0——0——5
prisms—0——0——2——11——7——3
Area —6——11——1——2——0——0

For Beer barrels.

pyramid 0—0—0—0—0—0—1—8
prisms—0—0—0—0—11—10—5
Area—0—2—3—8—1—8

For Ale Barrels.

pyramid 0-0-0-0-0-0-1-10-6
prisms—0-0-0-1-1-4-2-7-6
Area-—0-2-7-2-2-3

Note, If you make this (X) for 10: and this (Ẋ) for 11, you may set your duodecimals close without distinctions, in the same manner as you set Decimals, with only a point

be-

betwixt the integers, and duodecimals, thus may the numbers above for gallons be set

pyramid —0. 000 05
prisms — 0. 0 2Ẋ 73
Area ——— 6.Ẋ 1 2 00

Fifthly, For the Frustum of the Triangular pyramid, as standing on its lesser base, the numbers will be

For Gallons.

pyramid— .00022
prisms— .081
Area —— 10.

Now that you may see the manner of working when your figures are thus sate, I will work this so, to find the whole content of the Tun.

For Beer barrels.

pyramid — . 0000088
prisms —— . 0028400
Area ——– . 0340000

For Ale barrels.

pyramid—. 0000099
prisms—— . 0030460
Area ——— . 0390000

.00022 pyramid
8
———
.00154
6
———
.00880
.08100 prisms.
———
.08980
8
———
.5x540
6
———
2.Ẋ 2800
10.00000 Area.
———
12.Ẋ 2800
8
———
103.59400
6
———
620.x8000 whole content.

As ſtanding on its greater baſe,

For Gallons.

pyramid——. 00022
priſms———. 0X300
Area ——16. 14000

For Beer Barrels.

pyramid ——. 0000088
priſms——. 0035
Area ————. 5454

For Ale Barrels.

pyramid——. 0000099
priſms———. 003X16
Area ————. 606000

Sixthly, For the Fruſtum of the Polygonial pyramid, as ſtanding on its leſſer baſe, the numbers will be

For Gallons.

pyramid——. 0002016
priſms ——— . 0978160
Area —— 15. 4X60000

For Beer barrels.

pyramid ——. 0002016
priſms ——. 0032686
Area ———— . 5176

For

For Ale barrels.

pyramid —— . 000009069
prisms ———. 000125623
Area ———. 593X 30000

standing on its greater base, the numbers will be

For Gallons.

pyramid ——. 0002016
prisms- —— 0. 0 XX 94
Area- —— 23. 99

For Beer barrels.

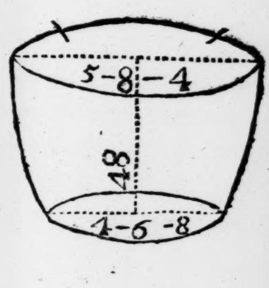
pyramid ——. 00000806
prisms ———. 003 XX 14
Area ———. 7 X 3

For Ale Barrels.

pyramid ——. 000009069
prisms ——. 003 X X
Area ———. 8X 1X6

Sixth Example, *Of a Spheroid.*

There is a Spheroidal Tun whose neter at top is 5 gall. 8 primes, 4″ diameter at bottom 4 gall. 6 prim. 8 seconds, both of the Circle are, and its depth 48 inches. *Q.* v many gallons may this Tun con- n?



As ſtanding on its greater baſe,

For Gallons,

pyramid——.00022
priſms——.0X300
Area ——16.14000

For Beer Barrels.

pyramid ——.0000088
priſms——.0035
Area ——.5454

For Ale Barrels.

pyramid——.0000099
priſms——.003X16
Area ——.606000

Sixthly, For the Fruſtum of the Polygonial pyramid ſtanding on its leſſer baſe, the numbers will be

For Gallons.

pyramid——.0002016
priſms ——.0978160
Area —— 15.4X60000

For Beer barrels.

pyramid ——.0002016
priſms ——.0032686
Area ——.5176

For Ale barrels.

pyramid ——. 000009069
prisms ———. 000125623
Area ———. 593X 30000

As standing on its greater base, the numbers will be

For Gallons.

pyramid ——. 0002016
prisms-——0. 0 XX 94
Area-—— 23. 99

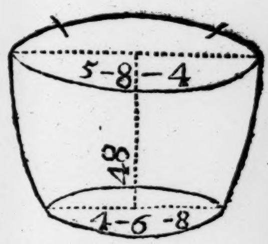
For Beer barrels.

pyramid ——. 00000806
prisms ———. 003 XX 14
Area ———. 7 X 3

For Ale Barrels.

pyramid ——. 000009069
prisms ———. 003 X X
Area ———. 8X 1X6

Sixth Example, *Of a Spheroid.*



There is a Spheroidal Tun whose diameter at top is 5 gall. 8 primes, 4″ and diameter at bottom 4 gall. 6 prim. and 8 seconds, both of the Circle square, and its depth 48 inches. Q. how many gallons may this Tun contain?

Note here, The ſlant ſides running not ſtraight, but having Curvitcure or Bulging, this Tun cannot be meaſured nor inched by the former Rules, therefore take this for a general Rule.

To find the whole Content, viz.

To the doubled Area of the greater Circle, add the Area of the leſſer, and multiply the ſum by the third part of the depth, and the product will be the whole Content.

And this
To Inch it, *viz.*

Find the Area of the Diameter at the top, and the Area of the diameter at 1 inch of the perpendicular from the top, and ſubduct one from the other and the difference will be the ſubducend, which taken from the Area of the firſt inch leaves the Area of the ſecond, and doubled and taken from the Area of the ſecond, leaves the Area of the third, trebbled and taken from the Area of the third leaves the Area of the fourth, and ſo you may go on gradually throughout the Tun, ſtill multiplying the difference by the increaſing inches and ſubducting the product from the Area on the laſt inch found.

If the Tun ſtands on its greater baſe, and you inch it downward, then the difference muſt be the Addend, and it muſt be gradually added, as the other is gradually to be ſubducted. And thus may a ſtanding Spheroidal Cask be inched, inching the higher part firſt by Addition home to the Boung diameter, and the lower part will be the ſame only the Area of the ſecond inch upward from the Bung, muſt be the Area of the ſecond inch downward, *&c.*

To

To find the whole Content.

Operation.

4 — 6 — 8 leſſer diameter.
4 — 6 — 8

18 — 2 — 8
2 — 3 — 4
2 — 3 — 4
9 — 1 — 4

20 — 9 — 0 — 5 — 4

5 — 8 — 4 greater diameter.
5 — 8 — 4

28 — 5 — 8
2 — 10 — 2
0 — 11 — 4 — 8
1 — 10 — 9 — 2

32 — 5 — 1 — 5 — 2
32 — 5 — 1 — 5 — 2
20 — 9 — 0 — 5 — 4

85 — 7 — 3 — 3 — 8
4

342 — 5 — 1 — 2 — 8
4

multiplyed by the ratio's of 16 the third of 48 inches the depth.

1369 — 8 — 4 — 10 — 8 anſwer in gallons.

The inching it is ſo plain as there needs no Example.

Note, the common way to inch any of the foregoing Tuns whoſe dimenſions differ at top and bottom (except the Spheroidal,) is this, *viz.*

Firſt, find the ſide of a ſquare equal in Area with the Area of the top, and another equal in Area with the Area of the bottom, (which may be found by extracting the ſquare Root, or by the Table, for the ſquare root of any Area is the ſide of a ſquare equal to it.) Then ſubtract the leſſer ſide ſo found from the greater, and then, as the whole depth is to the difference, ſo is any part of the depth (as ½ an inch, 1 inch, 2, 3, 4, &c. To a third number, which is called,

The { Addend or Subducend

And if you begin to inch at the leſſer end, it muſt be gradually added to the leſſer ſide throughout the whole depth of the Tun at every half inch, 1 inch, 2 inches, &c. according as you firſt begin. But if you begin at the greater end, you muſt accordingly ſubduct it.

And ſo you will have reduced the Tun into ſo many ſquares equal, then

Secondly, Having found all the ſides of the ſquares equal, by the Rule before given, you may take the ſide of the firſt inch, and the ſide of the ſecond inch, and add them together, and take ½ the total for the true ſide of the firſt inch, and the Area of that ſide will be the true Area for that inch: then take the ſide of the ſecond inch, and the ſide of the third inch and add them together, and half the total will be the true ſide for the ſecond inch, and the Area of that ſide will be the true Area for that inch; and ſo you may proceed throughoat the whole work, and add all the Area's toget her ſo found, and you will have the Tuns whole capacity.

And for an Example I will here again take the firſt Example of thoſe formerly inched, *viz.*

Ex-

Example.

A Tun in the form of a Prismoid whose dimensions are as followeth, *viz.*

above $\left\{\begin{array}{c}\text{gall.}\\ 12\\ 8\end{array}\right\}$ below $\left\{\begin{array}{c}\text{gall.}\\ 6\\ 4\end{array}\right\}$ depth 9 inches $\left\}\begin{array}{l}\text{Superficial.}\\ \text{How many gallons may this}\\ \text{Tun contain, and how ma-}\\ \text{ny at each inch.}\end{array}\right.$

differences $\left\{\begin{array}{c}6\\ 4\end{array}\right\}$ $\left.\begin{array}{c}3\\ 2\end{array}\right\}$ semy-differences.

$\left.\begin{array}{c}6\\ 4\end{array}\right\}$ differences multiplyed

24
3 the third of the depth.

72 solid content of the Pyramid.

6
4

24 Area of the parallelogram.
$\left.\begin{array}{c}12\\ 12\end{array}\right\}$ Area of the two prisms

48 the sum
9 depth

432 solid content of the parallelopipedon and 2 prisms.
72 solid content of the pyramid.

504 solid content of the Tun in gallons.

Now

Now to Inch it.

12
8

96 Area of the greater base.
24 Area of the lesser base.

gall.	′	″	‴	
9	9	6	10	the nearest side equal per Tab.
4	10	10	8	the nearest side equal per Tab.
4	10	8	2	difference

9) 4 — 10 — 8 — 2 (0 — 6′ — 6″ — 2‴ Addend or Subducend

4 — 9   ⁗ ⁗′ 10 — 8

· — 3 — 0

Now observe, I work for 1 inch, and begin with the lesser base, therefore I add 0 — 6 — 6 — 2 — 10 — 8, or which will do well enough 0 — 6 — 6 — 3 (the Fraction coming so near unto it) to 4 — 10 — 10 — 8, makes 5 — 5 — 4 — 11, for the side of a square equal to the Area of the upper side of the first inch, or the under side of the second, so adding it gradually, for every inch throughout the Tun, I find the side of the square for the last inch to be 9 — 9 — 6 — 11, which comes so near to the side found before for that inch, that I conclude my sides are rightly taken, therefore I proceed in the next place to find the mean sides, by adding the side equal to the under side of the first inch, and that equal to the upper side thereof together,

ther, and half their ſum is the true mean ſide for that inch, then I do the like by reſt, and having found all the mean ſides, I ſeek their Areas in the Table and againſt every mean ſide I ſet his proper Area, and then caſt them up, as you may ſee is done in the ſucceeding Table, where you may ſee it produces the ſame, for the whole content of the Tun, with the former caſting.

inch.	ſides equal.				mean ſides equal.					Areas.				
9	9	9	6	11										
8	9	3	0	8	9	6	3	9	6	90	7	9	0	9
7	8	8	6	5	8	11	9	6	6	80	7	6	0	9
6	8	2	0	2	8	5	3	3	6	71	2	3	6	9
5	7	7	5	11	7	10	9	0	6	62	4	1	6	9
4	7	0	11	8	7	4	2	9	6	54	1	0	0	9
3	6	6	5	5	6	9	8	6	6	46	3	9	5	4
2	5	11	11	2	6	3	2	2	0	39	2	9	0	4
1	5	5	4	11	5	8	8	0	6	32	8	11	1	4
	4	10	10	8	5	2	1	9	6	26	10	0	8	4
	underſide of the firſt inch.													

The ſame 504–0–2–7–1 again.

The

The Explanation of the Following Table, together with the several uses thereof.

THE Table is made by squaring of numbers, from one prime to 12 integers, including all the intermediate seconds, as you may plainly perceive by the Titles of the Columns, the Seconds are placed over the Columns, that are under the Title (Area of Squares or Circles) thus, 0 : 1 : 2 : 3 : 4 : 5 : 6 : 7 : 8 : 9 : 10 : 11. And the use of it is to find,

1. The Areas either of Squares or Circles, provided that the side of the square exceeds not 12 superficial square gallons, nor the diameter 12 of the Circle square gallons.

2. The square Root of any number under 144.

1. *Example.*

I would know the Area of a square whose side is 9 gall. 6 primes, 3 seconds, 9 thirds, 6 fourths; now looking (for the gallons and primes in the first Collumn under the Title, (sides of Squares, *&c.*) I find 9 gall. 6 prim. then looking up for the seconds, I find in the line above (which is the line for Seconds) 3 seconds. Then under 3 seconds and against 9 gall. 6 primes, I find 90 gall. 7 primes, 9 seconds, 0 thirds, 9 fourths, for the Area, which is the nearest square of the side given: for if you had taken your square under 4 seconds it would have been too much. But this comes near enough the Truth for common practice. The same order you are to observe to find the Areas of Circles.

2. *Example.*

I would know the square root of 94 gall. 9 primes, 6 sec. 0 thirds, 4 fourths, I look against this number in the first Column, and find 9 gall. 8 primes, then I look above for the seconds over the same number, and I find 10 seconds, so I conclude the square root to be 9 gall. 8 primes, 10 seconds.

Here place the large sheet Table.

Sides of Squares, Diameters or Sq. roots.		0					1					
Gal.	Pr	Gal.	P.	S.	T.	F.	Gal.	P.	S.	T	F.	G
	1			1					1	2	1	
	2			4					4	4	1	
	3			9					9	6	1	
	4		1	4				1	4	8	1	
	5		2	1				2	1	10	1	
0	6		3	0				3	1	0	1	
	7		4	1				4	2	2	1	
	8		5	4				5	5	4	1	
	9		6	9				6	9	6	1	
	10		8	4				8	5	8	1	
	11		10	1				10	2	10	1	
	0	1	00	0			1	0	2	0	1	
	1	1	2	1			1	2	3	2	1	
		9	[illegible]	[illegible]	[illegible]			140	2	3	8	1
		10	140	0	4			142	2	0	10	1 | 142 4
		11	142	0	1							

Areas of SQUARE

Sides of Squares, Diameters or Sq. roots.		0					1					2				
Gal.	Pr	Gal.	P.	S.	T.	F.	Gal.	P.	S.	T	F.	Gal.	P.	S.	T.	F.
0	1			1					1	2	1			1	4	4
	2			4					4	4	1			4	8	4
	3			9					9	6	1			10	0	4
	4		1	4				1	4	8	1		1	5	4	4
	5		2	1				2	1	10	1		2	2	8	4
	6		3	0				3	1	0	1		3	2	0	4
	7		4	1				4	2	2	1		4	3	4	4
	8		5	4				5	5	4	1		5	6	8	4
	9		6	9				6	9	6	1		7	0	0	4
	10		8	4				8	5	8	1		8	7	4	4
	11		10	1				10	2	10	1		10	4	8	4
1	0	1	00	0			1	0	2	0	1	1	0	4	0	4
	1	1	2	1			1	2	3	2	1	1	2	5	4	4
	2	1	4	4			1	4	6	4	1	1	4	8	8	4
	3	1	6	9			1	6	11	6	1	1	7	2	0	4
	4	1	9	4			1	9	6	8	1	1	9	9	4	4
	5	2	0	1			2	0	3	10	1	2	0	6	8	4
	6	2	3	0			2	3	3	0	1	2	3	6	0	4
	7	2	6	1			2	6	4	2	1	2	6	7	4	4
	8	2	9	4			2	9	7	4	1	2	9	10	8	4
	9	3	0	9			3	1	0	6	1	3	1	4	0	4
	10	3	4	4			3	4	7	8	1	3	4	11	4	4
	11	3	8	1			3	8	4	10	1	3	8	8	8	4
2	0	4	0	0			4	0	4	0	1	4	0	8	0	4
	1	4	4	1			4	4	5	2	1	4	4	9	4	4
	2	4	8	4			4	8	8	4	1	4	9	0	8	4
	3	5	0	9			5	1	1	6	1	5	1	6	0	4
	4	5	5	4			5	5	8	8	1	5	6	1	4	4
	5	5	10	1			5	10	5	10	1	5	10	10	8	4
	6	6	3	0			6	3	5	0	1	6	3	10	0	4
	7	6	8	1			6	8	6	2	1	6	8	11	4	4
	8	7	1	4			7	1	9	4	1	7	2	2	8	4
	9	7	6	9			7	7	2	6	1	7	7	8	0	4
	10	8	0	4			8	0	9	8	1	8	1	3	4	4
	11	8	6	1			8	6	6	10	1	8	7	0	8	4
3	0	9	0	0			9	0	6	0	1	9	1	0	0	4
	1	9	6	1			9	6	7	2	1	9	7	1	4	4
	2	10	0	4			10	0	10	4	1	10	1	4	8	4
	3	10	6	9			10	7	3	6	1	10	7	10	0	4
	4	11	1	4			11	1	10	8	1	11	2	5	4	4
	5	11	8	1			11	8	7	10	1	11	9	2	8	4
	6	12	3	0			12	3	7	0	1	12	4	2	0	4
	7	12	10	1			12	10	8	2	1	12	11	3	4	4
	8	13	5	4			13	5	11	4	1	13	6	6	8	4
	9	14	0	9			14	1	4	6	1	14	2	0	0	4
	10	14	8	4			14	8	11	8	1	14	9	7	4	4
	11	15	4	1			15	4	8	10	1	15	5	4	8	4
4	0	16	0	0			16	0	8	0	1	16	1	4	0	4
	1	16	8	1			16	8	9	2	1	16	9	5	4	4
	2	17	4	4			17	5	0	4	1	17	5	8	8	4
	3	18	0	9			18	1	5	6	1	18	2	2	0	4
	4	18	9	4			18	10	0	8	1	18	10	9	4	4
	5	19	6	1			19	6	9	10	1	19	7	6	8	4
	6	20	3	0			20	3	9	0	1	20	4	6	0	4
	7	21	0	1			21	0	10	2	1	21	1	7	4	4
	8	21	9	4			21	10	1	4	1	21	10	10	8	4
	9	22	6	9			22	7	6	6	1	22	8	4	0	4
	10	23	4	4			23	5	1	8	1	23	5	11	4	4
	11	24	2	1			24	2	10	10	1	24	3	8	8	4

Sides of Squares, Diameters or Sq. roots.		3					4					5			
Gal.	Pr	Gal.	P.	S.	T.	F.	Gal.	P.	S.	T.	F.	Gal.	P.	S.	T.
0	1			1	6	9			1	9	4			2	0
	2			5	0	9			5	5	4			5	10
	3			10	6	9			11	1	4			11	8
	4		1	6	0	9		1	6	9	4		1	7	6
	5		2	3	6	9		2	4	5	4		2	5	4
	6		3	3	0	9		3	4	1	4		3	5	2
	7		4	4	6	9		4	5	9	4		4	7	0
	8		5	8	0	9		5	9	5	4		5	10	10
	9		7	1	6	9		7	3	1	4		7	4	8
	10		8	9	0	9		8	10	9	4		9	0	6
	11		10	6	6	9		10	8	5	4		10	10	4
1	0	1	0	6	0	9	1	0	8	1	4	1	0	10	2
	1	1	2	7	6	9	1	2	9	9	4	1	3	0	0
	2	1	4	11	0	9	1	5	1	5	4	1	5	3	10
	3	1	7	4	6	9	1	7	7	1	4	1	7	9	8
	4	1	10	0	0	9	1	10	2	9	4	1	10	5	6
	5	2	0	9	6	9	2	1	0	5	4	2	1	3	4
	6	2	3	9	0	9	2	4	0	1	4	2	4	3	2
	7	2	6	10	6	9	2	7	1	9	4	2	7	5	0
	8	2	10	2	0	9	2	10	5	5	4	2	10	8	10
	9	3	1	7	6	9	3	1	11	1	4	3	2	2	8
	10	3	5	3	0	9	3	5	6	9	4	3	5	10	6
	11	3	9	0	6	9	3	9	4	5	4	3	9	8	4
2	0	4	1	0	0	9	4	1	4	1	4	4	1	8	2
	1	4	5	1	6	9	4	5	5	9	4	4	5	10	0
	2	4	9	5	0	9	4	9	9	5	4	4	10	1	10
	3	5	1	10	6	9	5	2	3	1	4	5	2	7	8
	4	5	6	6	0	9	5	6	10	9	4	5	7	3	6
	5	5	11	3	6	9	5	11	8	5	4	6	0	1	4
	6	6	4	3	0	9	6	4	8	1	4	6	5	1	2
	7	6	9	4	6	9	6	9	9	9	4	6	10	5	0
	8	7	2	8	0	9	7	3	1	5	4	7	3	6	10
	9	7	8	1	6	9	7	8	7	1	4	7	9	0	8
	10	8	1	9	0	9	8	2	2	9	4	8	2	8	6
	11	8	7	6	6	9	8	8	0	5	4	8	8	6	4
3	0	9	1	6	0	9	9	2	0	1	4	9	2	6	2
	1	9	7	7	6	9	9	8	1	9	4	9	8	8	0
	2	10	1	11	0	9	10	2	5	5	4	10	2	11	10
	3	10	8	4	6	9	10	8	11	1	4	10	9	5	8
	4	11	3	0	0	9	11	3	6	9	4	11	4	1	6
	5	11	9	9	6	9	11	10	4	5	4	11	10	11	4
	6	12	4	9	0	9	12	5	4	1	4	12	5	11	2
	7	12	11	10	6	9	13	0	5	9	4	13	1	1	0
	8	13	7	2	0	9	13	7	9	5	4	13	8	4	10
	9	14	2	7	6	9	14	3	3	1	4	14	3	10	8
	10	14	10	3	0	9	14	10	10	9	4	14	11	6	6
	11	15	6	0	6	9	15	6	8	5	4	15	7	4	4
4	0	16	2	0	0	9	16	2	8	1	4	16	3	4	2
	1	16	10	1	6	9	16	10	9	9	4	16	11	6	0
	2	17	6	5	0	9	17	7	1	5	4	17	7	10	10
	3	18	2	10	6	9	18	3	7	1	4	18	4	3	8
	4	18	11	6	0	9	19	0	2	9	4	19	0	11	6
	5	19	8	3	6	9	19	9	0	5	4	19	9	9	4
	6	20	5	3	0	9	20	6	0	1	4	20	6	9	2
	7	21	2	4	6	9	21	3	1	9	4	21	3	11	0
	8	21	11	8	0	9	22	0	5	5	4	22	1	2	10
	9	22	9	1	6	9	22	9	11	1	4	22	10	8	8
	10	23	6	9	0	9	23	7	6	9	4	23	8	4	6
	11	24	4	7	6	9	24	5	4	5	4	24	6	2	4

5			6					7				
S.	T.	F.	Gal.	P.	S.	T.	F.	Gal.	P.	S.	T.	F.
2	0	1			2	3				2	6	1
5	10	1			6	3				6	8	1
11	8	1		1	0	3			1	0	10	1
7	6	1		1	8	3			1	9	0	1
5	4	1		2	6	3			2	7	2	1
5	2	1		3	6	3			3	7	4	1
7	0	1		4	8	3			4	9	6	1
10	10	1		6	0	3			6	1	8	1
4	8	1		7	6	3			7	7	10	1
0	6	1		9	2	3			9	4	0	1
10	4	1		11	0	3			11	2	2	1
10	2	1	1	1	0	3		1	1	2	4	1
0	0	1	1	3	2	3		1	3	4	6	1
3	10	1	1	5	6	3		1	5	8	8	1
9	8	1	1	8	0	3		1	8	2	10	1
5	6	1	1	10	8	3		1	10	11	0	1
3	4	1	2	1	6	3		2	1	9	2	1
3	2	1	2	4	6	3		2	4	9	4	1
5	0	1	2	7	8	3		2	7	11	6	1
8	10	1	2	11	0	3		2	11	3	8	1
2	8	1	3	2	6	3		3	2	9	10	1
10	6	1	3	6	2	3		3	6	6	0	1
8	4	1	3	10	0	3		3	10	4	2	1
8	2	1	4	2	0	3		4	2	4	4	1
10	0	1	4	6	2	3		4	6	6	6	1
1	10	1	4	10	6	3		4	10	10	8	1
7	8	1	5	3	0	3		5	3	4	10	1
3	6	1	5	7	8	3		5	8	1	0	1
1	4	1	6	0	6	3		6	0	11	2	1
1	2	1	6	5	6	3		6	5	11	4	1
5	0	1	6	10	8	3		6	11	1	6	1
6	10	1	7	4	0	2		7	4	5	8	1
0	8	1	7	9	6	3		7	9	11	10	1
8	6	1	8	3	2	3		8	3	8	0	1
6	4	1	8	9	0	3		8	9	6	2	1
6	2	1	9	3	0	3		9	3	6	4	1
8	0	1	9	9	2	3		9	9	8	6	1
11	10	1	10	3	6	3		10	4	0	8	1
5	8	1	10	10	0	3		10	10	6	10	1
1	6	1	11	4	8	3		11	5	3	0	1
11	4	1	11	11	6	3		12	0	1	2	1
11	2	1	12	6	6	3		12	7	1	4	1
1	0	1	13	1	8	3		13	2	3	6	1
4	10	1	13	9	0	3		13	9	7	8	1
10	8	1	14	4	6	3		14	5	1	10	1
6	6	1	15	0	2	3		15	0	10	0	1
4	4	1	15	8	0	3		15	8	8	2	1
4	2	1	16	4	0	3		16	4	8	4	1
6	0	1	17	0	2	3		17	0	10	6	1
10	10	1	17	8	6	3		17	9	2	8	1
3	8	1	18	5	0	3		18	5	8	10	1
11	6	1	19	1	8	3		19	2	5	0	1
9	4	1	19	10	6	3		19	11	3	2	1
9	2	1	20	7	6	3		20	8	3	4	1
11	0	1	21	4	8	3		21	5	5	6	1
2	10	1	22	2	0	3		22	2	9	8	1
8	8	1	22	11	6	3		23	0	3	10	1
4	6	1	23	9	2	3		23	10	0	0	1
2	4	1	24	7	0	3		24	7	10	2	1

8					9					10					11				
Gal.	P.	S.	T	F.	Gal.	P.	S.	T.	F.	Gal.	P.	S.	T.	F.	Gal.	P.	S.	T.	F.
		2	9	4			3	0	9			3	4	4			3	8	1
		7	1	4			7	6	9			8	0	4			8	6	1
	1	1	5	4		1	2	0	9		1	2	8	4		1	3	4	1
	1	9	9	4		1	10	6	9		1	11	4	4		2	0	2	1
	2	8	1	4		2	9	0	9		2	10	0	4		2	11	0	1
	3	8	5	4		3	9	6	9		3	10	8	4		3	11	10	1
	4	10	9	4		5	0	0	9		5	1	4	4		5	2	8	1
	6	3	1	4		6	4	6	9		6	6	0	4		6	7	6	1
	7	9	5	4		7	11	0	9		8	0	8	4		8	2	4	1
	9	5	9	4		9	7	6	9		9	9	4	4		9	11	2	1
	11	4	1	4		11	6	0	9		11	8	0	4		11	10	0	1
1	1	4	5	4	1	1	6	6	9	1	1	8	8	4	1	1	10	10	1
1	3	6	9	4	1	3	9	0	9	1	3	11	4	4	1	4	1	8	1
1	5	11	1	4	1	6	1	6	9	1	6	4	0	4	1	6	6	6	1
1	8	5	5	4	1	8	8	0	9	1	8	10	8	4	1	9	1	4	1
1	11	1	9	4	1	11	4	6	9	1	11	7	4	4	1	11	10	2	1
2	2	0	1	4	2	2	3	0	9	2	2	6	0	4	2	2	9	0	1
2	5	0	5	4	2	5	3	6	9	2	5	6	8	4	2	5	9	10	1
2	8	2	9	4	2	8	6	0	9	2	8	9	4	4	2	9	0	8	1
2	11	7	1	4	2	11	10	6	9	3	0	2	0	4	3	0	5	6	1
3	3	1	5	4	3	3	5	0	9	3	3	8	8	4	3	4	0	4	1
3	6	9	9	4	3	7	1	6	9	3	7	5	4	4	3	7	9	2	1
3	10	8	1	4	3	11	0	0	9	3	11	4	0	4	3	11	8	0	1
4	2	8	5	4	4	3	0	6	9	4	3	4	8	4	4	3	8	10	1
4	6	10	9	4	4	7	3	0	9	4	7	7	4	4	4	7	11	8	1
4	11	3	1	4	4	11	7	6	9	5	0	0	0	4	5	0	4	6	1
5	3	9	5	4	5	4	2	0	9	5	4	6	8	4	5	4	11	4	1
5	8	5	9	4	5	8	10	6	9	5	9	3	4	4	5	9	8	2	1
6	1	4	1	4	6	1	9	0	9	6	2	2	0	4	6	2	7	0	1
6	6	3	5	4	6	6	9	6	9	6	7	2	8	4	6	7	7	10	1
6	11	6	9	4	7	0	0	0	9	7	0	6	4	4	7	0	10	8	1
7	4	11	1	4	7	5	4	6	9	7	5	10	0	4	7	6	3	6	1
7	10	5	5	4	7	10	11	0	9	7	11	4	8	4	7	11	10	4	1
8	4	1	9	4	8	4	7	6	9	8	5	1	4	4	8	5	5	2	1
8	10	0	1	4	8	10	6	0	9	8	11	0	0	4	8	11	6	0	1
9	4	0	5	4	9	4	6	6	9	9	5	0	8	4	9	5	6	10	1
9	10	2	9	4	9	10	9	0	9	9	11	3	4	4	9	11	9	8	1
10	4	7	1	4	10	5	1	6	9	10	5	8	0	4	10	6	2	6	1
10	11	1	5	4	10	11	8	0	9	11	0	2	8	4	11	0	9	4	1
11	5	9	9	4	11	6	4	6	9	11	5	11	4	4	11	7	6	2	1
12	0	8	1	4	12	1	3	0	9	12	1	10	0	4	12	2	5	0	1
12	7	8	5	4	12	8	3	6	9	12	8	10	8	4	12	9	5	10	1
13	2	10	9	4	13	3	6	0	9	13	4	1	4	4	13	4	8	8	1
13	10	3	1	4	13	10	10	6	9	13	11	6	0	4	14	0	1	6	1
14	5	9	5	4	14	6	5	0	9	14	7	0	8	4	14	7	8	4	1
15	1	5	9	4	15	2	1	6	9	15	2	9	4	4	15	3	5	2	1
15	9	4	1	4	15	10	0	0	9	15	10	8	0	4	15	11	4	0	1
16	5	4	5	4	16	6	0	6	9	16	6	8	8	4	16	7	4	10	1
17	1	6	9	4	17	2	3	0	9	17	2	11	4	4	17	3	7	8	1
17	9	11	1	4	17	10	7	6	9	17	11	4	0	4	17	11	10	6	1
18	6	5	5	4	18	7	2	0	9	18	7	10	8	4	18	8	7	4	1
19	3	1	9	4	19	3	10	6	9	19	4	7	4	4	19	5	4	2	1
20	0	0	1	4	20	0	9	0	9	20	1	6	0	4	20	2	3	0	1
20	9	0	5	4	20	9	9	6	9	20	10	6	8	4	20	11	3	10	1
21	6	2	9	4	21	7	0	0	9	21	7	9	4	4	21	8	6	8	1
22	3	7	1	4	22	4	4	6	9	22	5	2	0	4	22	5	11	6	1
23	1	1	5	4	23	1	11	0	9	23	2	8	8	4	23	3	6	4	1
23	10	9	9	4	23	11	7	6	9	24	0	5	4	4	24	1	3	2	1
24	8	8	1	4	24	9	6	0	9	24	10	4	0	4	24	11	2	0	1

3	6	12	3	0	12	3	7	0	1	12	4	2	0	4	12	4	9	0	9	12	5	4	1	4	12	5	11
	7	12	10	1	12	10	8	2	1	12	11	3	4	4	12	11	10	6	9	13	0	5	9	4	13	1	1
	8	13	5	4	13	5	11	4	1	13	6	6	8	4	13	7	2	0	9	13	7	9	5	4	13	8	4
	9	14	0	9	14	1	4	6	1	14	2	0	0	4	14	2	7	6	9	14	3	3	1	4	14	3	10
	10	14	8	4	14	8	11	8	1	14	9	7	4	4	14	10	3	0	9	14	10	10	9	4	14	11	6
	11	15	4	1	15	4	8	10	1	15	5	4	8	4	15	6	0	6	9	15	6	8	5	4	15	7	4
	0	16	0	0	16	0	8	0	1	16	1	4	0	4	16	2	0	0	9	16	2	8	1	4	16	3	4
	1	16	8	1	16	8	9	2	1	16	9	5	4	4	16	10	1	6	9	16	10	9	9	4	16	11	6
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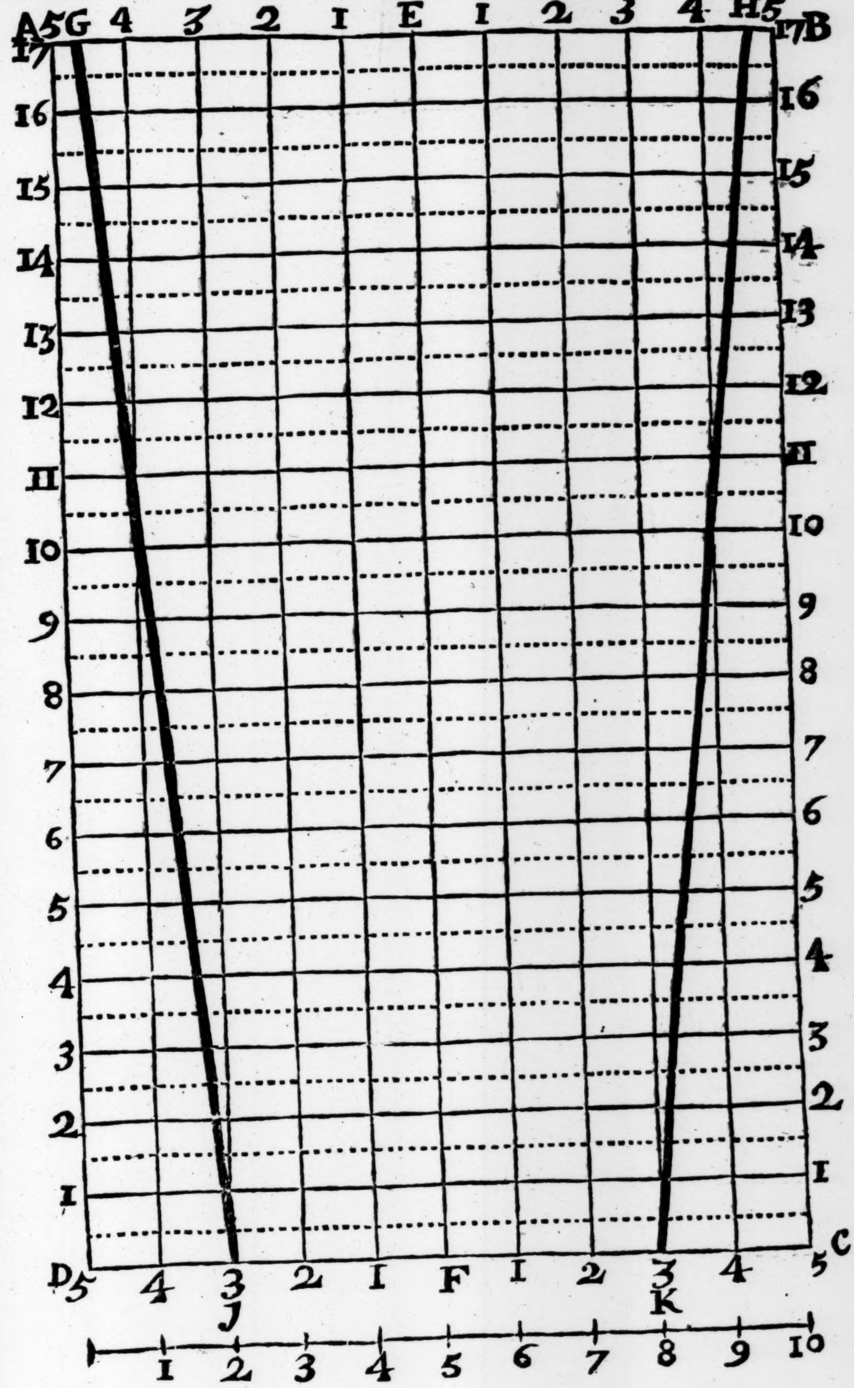
56	1	9	0	1
57	4	9	10	1
58	8	0	8	1
59	11	5	6	1
61	3	0	4	1
62	6	9	2	1
63	10	8	0	1
65	2	8	10	1
66	6	11	8	1
67	11	4	6	1
69	3	11	4	1
70	8	8	2	1
72	1	7	0	1
73	6	7	10	1
74	11	10	8	1
76	5	3	6	1
77	10	10	4	1
79	4	7	2	1
80	10	6	0	1
82	4	6	10	1
83	10	5	8	1
85	5	2	6	1
86	11	9	4	1
88	6	6	2	1
90	1	5	0	1
91	8	4	10	1
93	3	8	8	1
94	11	1	6	1
96	6	8	4	1
98	2	5	2	1
99	10	4	0	1
101	6	4	10	1
103	2	7	8	1
104	11	0	6	1
106	7	7	4	1
108	4	4	2	1
110	1	3	0	1
111	10	3	10	1
113	7	6	8	1
115	4	11	6	1
117	2	6	4	1
119	0	3	2	1
120	10	2	0	1
122	8	2	10	1
124	6	5	8	1
126	4	10	6	1
128	3	5	4	1
130	2	2	2	1
132	1	1	0	1
134	0	1	10	1
135	11	4	8	1
137	10	9	6	1
139	10	4	4	1
141	10	1	2	1
143	10	0	[illegible]	[illegible]

But he that pleaſe may make an inſtrument ready on all occaſions whereby to find the mean ſides of ſquares equal for every inch of any of the aforeſaid Tuns differing in their dimenſions at top and bottom, which may be made on Braſs. The manner whereof I have laid down in the following Figure, where let A B C D repreſent a piece of Braſs, E F the perpendicular, divided into 17 equal parts repreſenting inches of the Tuns depth, and the lines drawn from ſide to ſide marked with figures 1, 2, 3, 4, *&c.* are to find the ſides of ſquares equal, and the lines drawn in the middle between them are to find the mean ſides of ſquares equal to every inch of the depth: G I and H K are lines repreſenting the ſides of the Tun, which muſt be thin pieces of Braſs, made to ſlide to and fro at top and bottom, ſo as they may be placed at what figure you pleaſe, both on high and below. And note, the lines A B and D C muſt be Duodecimally divided, and thus having fitted your ſelf with ſuch an inſtrument, you may readily find the mean ſides of ſquares equal, to every inch of the inquired Tuns depth in this manner. As ſuppoſe the ſide of the ſquare equal in Area to the Area of the bottom of your Tun, be 6 ſuperficial gallons, then place G I and H K at 3 and 3 at the bottom, 3 being the half of 6, and the ſide of the ſquare equal in Area to the Area of the top be 9 ſuperficial gallons, and the depth 16 inches, then let G I and H K interſect the line 16 and 16 in the lines 4 half and 4 half on each ſide of the perpendicular, 4 half being the half of 9, then ſhall the lines in the middle of every repreſented inch, be the mean ſide of the ſquare equal in Area to the Area of every inch throughout the Tun, the Areas whereof may be found by the Table: and you may take the mean ſides in your Compaſſes, and find them on a ſcale of equal parts to be made by the line D C under the inſtrument. And though I have made it but for 17 inches deep, you may divide it again into 34 or 68, or ſo as to make it ſerve for any uſual depth.


A 5 G 4 3 2 1 E 1 2 3 4 H 5 17 B
17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
C
D 5 4 3 2 1 F 1 2 3 4 5
J
K
1 2 3 4 5 6 7 8 9 10

To find the Content of the Liquor that covers a Coppers rising Crown.

Take the Diameter at the top of the Crown by the Circle ſquare, and the diameter in the bottom of the Copper and add half the difference to the leſſer diameter, then ſquare that mean diameter, and multiply it by half the Altitude.

If the higher part of the Copper be Conical or Spheroidal, you muſt deal with it according to former rules for ſuch Tuns reſpectively, both to find its whole Content, and inching it. And having found the Content of the upper part, add the Content of the Liquor that covers the Crown thereunto, and you will have the whole Content of the Copper.

But if the Coppers Crown turns downward, it is the Fruſtum or ſection of a Globe, and then take this Rule to find the Content of the falling Crown.

Viz.

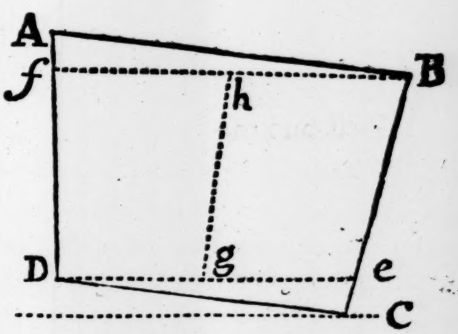
Firſt find the Globes Axe, thus, divide the ſquare of the ſemi-diameter by the Fruſtums Altitude in inches, and the quote will be the remainder of the Globes Axe, or Altitude of the greater Fruſtum, which added with the altitude of the leſſer, will be the Globes Axe ſought.

Then by the Diameter at the Fruſtums baſe, find the Area of the Circle by the Circle ſquare, which multiply by the leſſer Fruſtums altitude, reſerving the product. Then to half the altitude of the greater Fruſtum, add $\frac{1}{6}$ of the altitude of the leſſer, and multiply the ſum by the former product, and divide this laſt product by the altitude of the greater Fruſtum, and the quotient will be the content of the leſſer Fruſtum ſought.

To

To find the Content of a Mash Tun, whether bounded with Right Lines, or Circular.

There can be no certain Rule in taking the depth of the Goods by reaſon of the difference in grinding, but this may come near enough the truth, whereby a fraud may often be diſcovered, *viz.* Find the Area or Superficial content of one inch of the depth by the Rules afore given, (having reſpect to the form,) and add a fourth thereof thereunto, and multiply the total by the inches of the depth, and you will have the ſolid content in gallons. Then to turn the gallons into Quarters and Buſhels, 8 Gallons do make the buſhel, and 8 buſhels or 64 gallons do make a Quarter, therefore take the 8th. of the gallons, and the product will be buſhels, then take the 8th. of the buſhels and the product will be Quarters.



If any of the aforeſaid Tuns (differing in their dimenſions at top and bottom) are leaning, as the Figure A B C D repreſents, if you ſtretch a cord from D to *e*, and apply a ſmall pocket level thereunto, moving the end of the cord that is towards *e* up or down, untill you find it level, then ſhall D *e* be the level, *viz.* the line of the Superficies of the liquor that ſhall cover the bottom; then lay off C *e* from A to *f*, and if you ſtretch a cord from B to *f*, then will *g h* be the depth of the liquor when the Tun is full, and you are to take your other dimenſions from *f* to B and from D to *e*, both to find its whole content, and to inch the ſame, adding the liquor contained in D *e* C (*viz.* the liquor that covers the

bottom) thereunto; and the trueſt and ſureſt way to find it, is to pour in liquor until the bottom be juſt covered, and meaſure it as you pour it in, as I have already noted.

To find the Allowances for Beer and Ale, according to the rate of 3 barrels in 23 for Beer, and 2 into 22 for Ale.

Firſt for Beer, divide the number of barrels by 7 integ. 8 primes, and the quotient gives the Allowance.

Example.

I demand the Allowance for 324 barrels of Beer.

```
7—8) 324         Facit   0—01—0
     ——————— (42—3 ———————————————
      32—0               0— 7—8
     ———————
      1—4                Barr.
     ———————         viz. 42 and 1/4
     17—4
     ——————————
      2—0—0
     ——————————
      .—1—0
```

Secondly for Ale. Take the 11th. of the number of barrels, and you will have the Allowance.

Example.

I demand the Allowance for 324 barrels of Ale.

```
  11) 324
     ———————
Barrels 29 — 05′ — 5″ for the Allowance.
```

To find what the Exciſe of any number of barrels comes to at any Exciſe the barrel.

Multi-

Multiply the barrels by the ſhillings and pence of the Exciſe, Duodecimally, then cut off the laſt figure to the right hand of the ſhillings, and take half of the reſt, and then your anſwer will be in pounds, ſhillings and pence.

Example.

889 barrels at 2 s. 9 d. per barrel.

2—9

1778
444 — 6
222 — 3

2441|4 — 9
122 — 4 — 9

	l.	s.	d.
Facit	122	4	9

If there be any primes, ſeconds, *&c.* take ſuch parts of the Exciſe, per barrel as the primes, ſeconds, *&c.* do make,

As if there were 4 primes, 8 ſeconds,
at 2 s. 9 d. per barrel, take

$\frac{1}{3}$—0—11 for 4 primes
$\frac{1}{6}$ of that 0 — 1 — 10 for 8 ſeconds they being the $\frac{1}{6}$ of 4 primes.

Facit 1 s. — 00 — 10, *viz.* 1 s. 00 d. and $\frac{10}{12}$ of 1 d. which makes $\frac{3}{4}$ and $\frac{1}{12}$ of a penny more.

Duodecimal Cask-Gauging.

Cask-Gauging Duodecimally performed.

IF a Cask be taken as the Fruſtum of a Spheroid (under which capacity Casks have until of late years been too commonly received) Mr. *Oughtreds* Gauging rod (as full) gives the content. But by Experience, and common practiſe ſome Casks are found differing in form, and are more parabolical than Spheroidal. Therefore that you may know what form any Cask is of at the firſt view, I ſhall give you theſe following Theorems (which I gratefully acknowledge to have received of my ingenious Friend Mr. *Marmaduke Hodgeſon* of *Croutched Fryers*, *London*;) whereby you may project at your leiſure, the middle Fruſtum of a Spheroid, the middle Fruſtum of a parabolical ſpindle, and the middle Fruſtum of two equal parabolical Conoids, by calculating their diameters betwixt Bung and Head, and ſo in a little time come to the knowledge of each form.

And before I lay down the Theorems, I will propoſe the dimenſions of a Cask as common to all the ſaid three forms, whereby to proportion them, *viz.*

Bung Diameter ———	31	Inches.
Head Diameter ———	25	
Length ——— ———	50	

Now

Now First, To project the form of the middle frustum of a Spheroid, by finding all or any of the Diameters betwixt bung and head.

Theorem.

As the square root of the difference of squares of Bung and head diameters, is to the Bung diameter, so is the length to the Spheroids Axis, from the square of which Axis, subtract 4 times the desired distance from the Bung, and extract the square root of the remain, Then, as the Axe is to this square-root, so is the bung Diameter to the Diameter at the desired distance.

Secondly, To project the form of the middle frustum of a parabolical Spindle, by finding all or any of the Diameters betwixt Bung and Head.

Theorem.

As the difference betwixt Bung and head diameters to the bung diameter, so is the square of the length to the square of the Axis, the square-root whereof will be the Axis.

Then divide a fourth of the square of the Axe by the semi-bung diameter, and you will have a number to be reserved: To and from the semi-axe, add and subtract the desired distance from the bung, then multiply the sum by the remainder, and divide the product by the reserved number, the quotient will be the semi-diameter, at the distance desired.

Thirdly, To project a Cask in the form of the middle frustum of two equal parabolick Conoids, by finding all, or any of the diameters betwixt bung and head.

Theorem.

Divide a fourth of the difference of ſquares betwixt bung and head diameters by the ſemi-length, and you will have a number to be reſerved, by which if you divide the fourth of the ſquare of the bung diameter, you will have the Axis: from the Axis ſubtract the deſired diſtance from the bung, and multiply the remain by the reſerved number, and the ſquare root of the product will be the ſemi-diameter at the diſtance deſired.

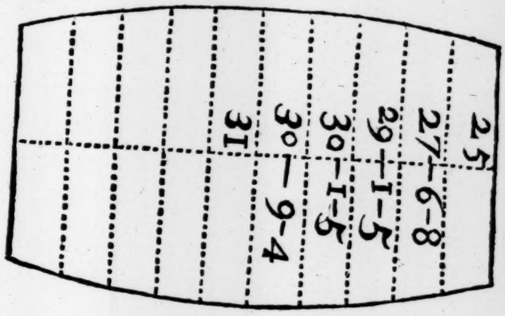
And according to theſe Theorems, (to help you in your practice) I have calculated Diameters at 5 inches diſtance one from the other throughout the Cask, for all the three forms, proportioned by the dimenſions of the Cask firſt propoſed.

First,

First, For the middle frustum of a Spheroid.

inch. ′ ″
Axis ———84-6-11
inches.

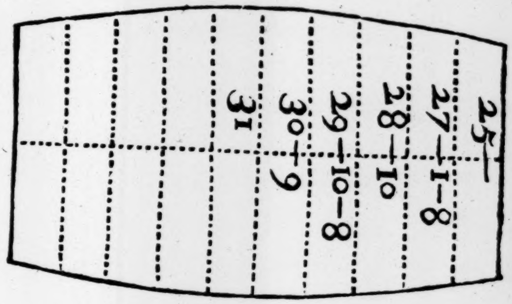
diam. from the bung at —
5—30-9-04
10—30-1-05
15—29-1-05
20—27-6-08
25—25-0-00



Secondly, For the middle frustum of a Parabolical spindle.

inch. ′ ″
Axis ——-113- 7-10
inches.

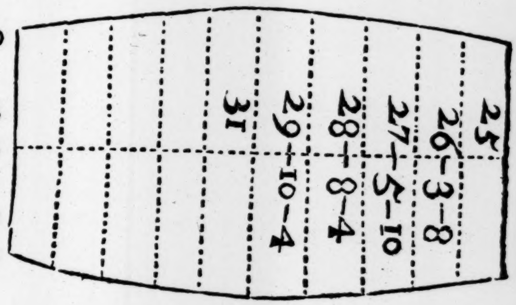
Semi-diam. from the bung at—
5—15- 4-06
10—14-11-04
15—14-05-00
20—13-06-10
25—12-06-00



Thirdly, For the middle frustum of two equal parabolical Conoids.

inch. ′ ″
Axis ——— 71-06-00
inches.

Semi-diam. from the Bung at—
5—14-11-02
10—14-04-02
15—13-08-11
20—13-01-10
25—12-06-00



Note, The difference of these 3 forms would more plainly appear if they were larger projected.

In the next place I shall lay down a plain, and easie method to find the whole contents of Casks under these 5 considerations, *viz.*

As —
1. Cylindrical.
2. Conical.
3. Spheroidal.
4. The middle frustum of a parabolical spindle.
5. The middle frust. of 2 equal parabolical Conoids

And for the easier finding their contents I have made a scale to find the Area in Ale or Wine gallons of any Circle, whose diameter exceeds not 6 foot, onely by laying it on the diameter, which I call the Circle Gauge. But before I shew you the use thereof, I think it requisite to let you know the manner and method of making it, that you may the better understand the ground and reason of working by it. Therefore for the Circle Gauge for Ale measure, *viz.* to find the length of every gallon on any diameter.

You may remember, you have been taught that as 14 is to 11 so is the square of the diameter to the content in inches, which divided by 282 (the number of inches contained in an Ale gallon,) gives the content in gallons, therefore

As 11 is to 14, so are 282 inches to that number, that shall be the Gauge number, *viz.* 359, but if you work it, you shall find the Gauge number nearer the truth to be 358 10 – 11, by which number if you divide the square of the diameter, the quotient gives the gallons contained in the Circle at one inch deep. Therefore

Again,

Again, The ſquare root of 358 —— 10 —— 11 muſt be the length of one gallon, *viz.* of the firſt gallon to be placed on the Circle Gauge. Then for the ſecond gallon, double 282 and find the Gauge number thereof as before, and the ſquare root thereof will be the length of the ſecond gallon, and ſo you may do *ad infinitum*, ſtill adding 282 to the former number.

And the ſame method muſt be followed if you deſcend to the prime of a gallon, or the 12th. of a prime, *&c.* But here Note, in ſtead of multiplying by 14, and dividing by 11, you may add $\frac{3}{11}$ to the Area, (14, being $\frac{3}{11}$ more than 11) and the ſquare root of the total gives the diameter. And according to this method I have made the enſuing Tables, to be placed on a ſcale, *viz.* one for Ale meaſure at 282 inches to the gallon, another for Wine meaſure, at 231 inches to the gallon.

For A L E.

		Length of the Diameter.						Length of the Diameters.			
gall.	pri.	inch.	pri.	″	‴	G.	Pr.	in.	Pr	S.	T.
	3	9	05	07	11						
	6	3	11	01	01						
	9	3	00	01	06						
1	0	18	11	04	05	8	0	3	5	6	4
	3	2	2	10	0						
	6	2	0	3	2		6	1	7	9	6
	9	1	10	3	8						
2	0	7	10	2	0	9	0	3	3	0	1
	3	1	7	6	0						
	6	1	6	5	4		6	1	6	8	3
	9	1	5	6	6						
3	0	6	0	3	1	10	0	3	0	10	8
	3	1	4	1	0						
	6	1	3	5	7		6	1	5	9	1
	9	1	2	11	2						
4	0	5	0	11	0	11	0	2	11	0	9
	3	1	1	11	11						
	6	1	1	11	1		6	1	4	11	8
	9	1	0	11	1						
5	0	4	5	8	0	12	0	2	9	6	8
	3	1	0	6	10						
	6	1	0	2	11		6	1	4	2	10
	9	0	11	10	9						
6	0	4	0	6	3	13	0	2	8	1	11
	3	0	11	5	9						
	6	0	11	3	1		6	1	3	7	4
	9	0	11	0	6						
7	0	3	8	7	5	14	0	2	6	11	3
	3	0	10	7	9						
	6	0	10	5	7		6	1	3	1	1
	9	0	10	3	6						
						15	0	2	5	8	4

For WINE.

		Length of the Diameters.						Length of the Diameters.			
gall.	pri.	in.	P.	S.	T.	gall.	pri.	in.	P.	S.	T.
	3	8	6	10	6						
	6	3	6	7	5						
	9	2	8	8	4						
1	0	17	1	9	1	8	0	3	1	7	0
	3	2	0	0	4						
	6	1	9	11	6		6	1	5	10	11
	9	1	8	1	11						
2	0	7	0	11	7	9	0	2	11	3	8
	3	1	5	7	10						
	6	1	4	8	4		6	1	4	10	11
	9	1	3	10	7						
3	0	5	5	4	9	10	0	2	9	4	8
	3	1	2	6	8						
	6	1	2	0	0		6	1	4	2	1
	9	1	1	6	2						
4	0	4	7	1	8	11	0	2	7	9	1
	3	1	0	7	11						
	6	1	0	3	9		6	1	3	3	11
	9	0	11	11	4						
5	0	4	0	6	10	12	0	2	6	4	1
	3	0	11	4	4						
	6	0	11	1	2		6	1	2	8	5
	9	0	10	10	1						
6	0	3	7	11	0	13	0	2	5	1	3
	3	0	10	4	8						
	6	0	10	2	3		6	1	2	1	8
	9	0	9	11	11						
7	0	3	4	4	7	14	0	2	4	0	1
	3	0	9	7	7						
	6	0	9	5	8		6	1	1	7	6
	9	0	9	3	9						
						15	0	2	3	0	3

To make the ſcales by the Tables, for the firſt 3 primes of a gallon in that for Ale (obſerving the ſame order for that for Wine) take from a ſcale of inches (divided into twelves, *viz.* primes, ſeconds, thirds) 9 inches 5 — 7 — 11 in your Compaſſes, and lay it on your ſcale, then for 6 primes, take 3 inches, 11 — 1 — 1, and lay it from the former, then for 9 primes take 3 — 0 — 1 — 6, and lay it from the 6 primes, then for one gallon, take 18—11—4 — 5 and lay it from the beginning of your ſcale, then for the next 3 primes take 2—2—10, and lay it from 1 gallon, then lay 2 — 0 — 3 — 2 from the laſt 3 primes for 6 primes, then 1 — 10 — 3 — 8 from thence for 9 primes, then for 2 gallons take 7 — 10 — 2, and lay it from 1 gallon, and ſo do for all the reſt.

Now for as much as the dimenſions muſt be the firſt thing known, before the content can be found, I ſhall therefore ſhew how by ſome of the dimenſions to find the reſt, if any obſtructions prohibit the taking of all.

I. Having the Bung-diameter, Head-diameter, and Diagonal to find the Casks length.

The Rule.

Firſt ſubduct the ſemi-difference of diameters from the Bung-diameter, and ſquare the remainder, and ſubduct that ſquare from the ſquare of the Diagonal, and the ſquare root of the remain is the Casks ſemi-length.

Example.

Let the Bung-diameter be ——— ——— 29 inches
Head-diameter ——— ——— — 23
Diagonal ——— ——— ——— 35 — 3′
Semi-difference of diameter ——— 3

Q. What is the Casks length.

29

29 bung diameter.
23 Head diameter.

·6 difference.

3 ſemi-difference

26 remain
26

156
52

676 ſquare of the remain.

35 — 3 diagonal
35 — 3

175
1058 — 9
8 — 9 — 9

1242 — 6 — 9 ſquare of the diagonal.
676 — 0 — 0

566 — 6 — 9 ſquare of the ſemi-length

23 — 9 — 6 ſquare-root. Semi-length

166
43

37 — 6 — 9
3 — 10 — 9

2 — 0 — 1 — 0 — 0
3 — 11 — 6 — 6

: — 3 — 9 0

23 — 09 — 6
23 — 09 — 6

47 — 7 — · length of the Cask.

II. Having

II. Having the bung-diameter, diagonal and length to find the head-diameter.

The Rule.

From the quadrupled ſquare of the diagonal ſubduct the ſquare of the length, and the ſquare-root of the remainder is equal to the ſumme of the bung diameter, and one head diameter, wherefore ſubducting the bung-diameter, the remain will be the head-diameter.

Example.

Let the bung-diameter, diagonal and length be the ſame with the former.

1242 — 6 — 9 ſquare of the diagonal.
4

4970 — 3 — 0 quadrupled ſquare of the diagonal.

47 — 7 length.
47 — 7

329
1880
23 — 6
03 — 11
23 — 9 — 6
3 — 11 — 7

2264 — 2 — 1 ſquare of the length.

52 — 0 — 2
29 — 0 — 0 bung-diameter,

23 — 0 — 2 head-diameter.

4970 — 3 — 0 quad. ſquare of the diagonal.
2264 — 2 — 1 ſquare of the length.

2706 — 0 — 11 remain.
52 — 0 — 2 ſquare-root.

206
102

02 — 0 — 11 — 0 — 0
8 — 8 — 0

III. Hav-

III. Having the Head-diameter, Bung-diameter, and the length, to find the Diagonal.

The Rule.

Subduct the ſemi-difference of the diameters at bung and head, from the bung-diameter, and to the ſquare of the remain add the ſquare of the ſemi-length, and the ſquare-root of their ſum will be the diagonal.

Example.

Let the dimenſions be the ſame with the former.

23	9	6 ſemi-length.		
23	9	6		
166	6	6		
499	7	6		
47	7	0		
11	10	9		
5	11	4	6	
0	11	10	9	
566	0	6	3	

```
 29 bung-diameter
 23 head-diameter
 ---
  6
 ---
  3 semi-difference
 ---
 26 remain
 26
 ---
156
52
 ---
676
566 —— 0 —— 6 —— 3
 ---
1242 —— 0 —— 6 —— 3
 ---
35 —— 2 —— 11 squ.root for the diagonal which is very
                     near the same again.
 ---
342
 65
 ---
 17 —— 0 —— 6
  5 —— 10 —— 2
 ---
  5 —— 4 —— 2 —— 3 —— 0
       5 —— 10 —— 4 —— 11
 ---
       • —— 7 —— 8 —— 11
```

Now I come to shew you the use of the Circle Gauge for Wine measure, (and the same method must be observed by the Circle Gauge for Ale measure,) in finding the whole content.

First,

First, *Of a Cylindrical Cask.*

The Rule.

Multiply the Area of the bung or head Circle by the length, and you have the Content.

Example.

Let the Area of the bung or head Circle (found by the Circle Gauge for Wine) be 2 gallons, 10 primes, and length 36 inches. How many Wine gallons doth it contain?

	gall.	′	
	2	10	multiplyed by the ratio's of 36.
	17	0	
Facit	102	0	

Secondly, *If a Cask be taken as the middle frustum of two Cones abutting upon one common Base*, &c.

The Dimensions.

	gall.	′	″
Bung	2	8	4
Head	1	9	3
Length	48 inches		

The Rule.

To one third of the Area of the bung Circle, add one third of the Area of the head Circle, and to their summ add the semi-sum thereof, and from that sum subduct the $\frac{1}{6}$ of the Area of the differing inches between the diameters of head and bung, and multiply the remain by the Casks length.

Example.

0 — 10 — 9 — 4 the third Area of bung
0 — 07 — 1 — 0 the third Area of head

1 — 05 — 10 — 4 Their sum
0 — 08 — 11 — 2 Semi-sum

2 — 02 — 9 — 6
2 — 8 the $\frac{1}{8}$ of the Area of 6 inch. diff. between the diameters.

2 — 02 — 6 — 10 Area of the mean Circle.
48 the length.

17 — 8 — 6 — 8

106 — 3 — 4 — 0 *Answ. viz.* 106 gall. 1 quart and one fourth of a pint.

Thirdly, *Of a Cask taken as the frustum of a Spheroid, cut with two planes parallel, each plane bisecting the Axis at right Angles.*

The Rule.

To two thirds of the Area of the bung-circle, add one third of the Area of the head-circle, and multiply their sum by the Casks length, and the product will be the Casks whole Content.

Example.

Let the Area of the bung-circle (found by the circle-gauge for Wine-measure) be 2 gall. 8 primes, 4 sec. and the Area of the head-circle be 1 gallon, 9 primes, 3 seconds, and the length 48 inches. *Q.* What is the Casks whole Content in Wine gallons ?

$\frac{1}{3}$) 2 — 8

$\frac{1}{3}$) 2 — 8 — 4 bung diameter. gall.

0 — -10 — 9 — 4
0 — -10 — 9 — 4

1 — -09 — 6 — 8 the $\frac{2}{3}$ of the Area of the Bung.
0 — -07 — 1 — 0 the $\frac{1}{3}$ of the Area of the Head.

2 — -04 — 7 — 8. Area of the mean Circle.
48

14 — -03 — -10 — 0

114 — -06 — 8 — 0 Answer.

$\frac{1}{3}$) 1 — 9 — 3 head-diam.

0 — 7 — 1 the $\frac{1}{3}$ of the Area.

That is 114 gall. 2 quarts and almost half a pint.

Fourthly, *To find the solid Content of a Cask when taken as the middle Frustum of a Parabolical spindle, the dimensions as before.*

The Rule.

To the $\frac{2}{3}$ of the Area of the bung circle, add the third of the Area of the head-circle, and to their sum add the half thereof, and the $\frac{1}{10}$ of the difference of Areas of head and bung, and multiply their sum by the length, and the product Answers.

$\frac{1}{3}$) 2 — 8 — 4 bung

0 - -10 — 9 — 4 $\frac{1}{3}$ thereof.
0 — -07 — 1 — 0 $\frac{1}{3}$ of head

1 - -05 - -10 — 4
0 — -08 — -11 — 2
1 — 1 — 3

2 — -03 - -10 — 9 Area of mean Circle.

2 — 8 — 4
1 — 9 — 3

$\frac{1}{10}$) 0 — 11 — 1

1 — 1 — 3

2 —— 3 —— 10 —— 10
48

13 —— 11 —— 5 —— 0

111 —— 7 —— 4 —— 0 Anſwer.

That is 111 gall. 2 quarts, 1 pint and almoſt half.

Fifthly, *To find the ſolid Content of a Cask if taken as the Fruſtum of a parabolical Conoid.*

The Dimenſions as before.

The Rule.

To a third of the Area of the bung circle add a third of the Area of the head-circle, and to their ſum add the ſemiſum thereof, and multiply that ſum by the Casks length, and the product will be the anſwer.

Example.

0 —— 10 —— 9 —— 4 the third Area of bung.
0 —— 07 —— 1 —— 0 the third Area of head.

1 —— 05 —— 10 —— 4 their ſum.
0 —— 08 —— 11 —— 2 ſemi-ſum.

2 —— 02 —— 9 —— 6 Area of the mean circle.
48 the length.

13 —— 04 —— 9 —— 0

107 —— 02 —— 0 —— 0 anſwer, *viz.* 107 gall. 1 pint $\frac{1}{3}$.

Now

Now the remaining Liquor, or the vacuity, or wants, may be found under these two considerations, *viz.*

1. A Cask lying with the Axe parallel to the Horizon.
2. A Cask standing on the head with the diameters parallel to the Horizon.

And for a Cask as lying, I have made a Segment line, which is done by the dividing a circle by the Chord lines, whose diameter is unity, divided into 1728 equal parts, very proper for a Cylinder, and with a very little trouble may be practicable for a Spheroidal, a parabolical and Conical Cask also. For if any please to try, he shall find not above one gallon difference upon any segment throughout the whole bung-diameter, between the operations by this scale and those by any Table calculated, or scale fitted for a Spheroidal Cask, (upon the mediety there can be none); and the difference being near about a gallon more upon the lesser segment, and the same less upon the greater segment, It is but to add or subtract a gallon as you have occasion, and it will answer as near the truth as any other practical way whatsoever. And it may do as well for a parabolical Cask, for the difference between the segments of a Cask Spheroidal, and a Cask parabolical, is not above 2 or 3 tenths at the most, which is very inconsiderable in practice. though that may easily be accomodated too.

How ever, I were intended to have made a scale for a Spheroidal, and another for a parabolical Cask, as lying and as standing, after the same manner with this, but several hindrances have intervened, so that I wanted time to perform it; but if it shall please God to give me Life and Health, I intend to do it in a little time, and shall so demonstrate the same, as the reason thereof shall plainly appear to any ordinary capacity, and those for lying Casks shall be made to Answer as well when the Liquor cuts the sides as the heads, whereas this, (as I conceive) and all other wayes yet extant, answers onely as long as the Liquor cuts the heads.

And yet at present for a standing Cask I have directed Mr. *Isaac Carver* of *Redriff*, to make a scale for $\frac{2}{3}$ and $\frac{1}{3}$ Area both for Ale and Wine gallons, (which may be placed on the same scale with the Circle-Gauge,) and if a line of inches be ran by the side of them duodecimally divided, you may soon find the mean Area of the segment, whether wet or dry, by the Rules hereafter given, which found and multiplyed by the wet or dry inches, you will have the content of the segment. And if you cannot take the diameter of the Liquors superficies, you may soon find it by the circumference square, by girting it, as you have been taught before.

First, *Of a Cask as Lying.*

And first of a Cylinder.

Proposition. 1.

Suppose a Cylinder lying with its diameters perpendicular to the Horizon, and having some Liquor remaining in it, to find the content of the said Liquor in gallons?

In this Proposition there are four things to be known, *viz.*

The diameter at the bung or head ——— 29 inches.
The length of the Cylinder ——— 48
The depth of the Liquor ——— 12

And the whole content in gallons ——— $137 - 3' - 8''$

As the bung diameter to the wet, or dry inches.
So is 1 to a number, which sought in the line of inches, just against it in the line of segments will be found a number, by which if you multiply the whole content, the product will be the remaining Liquor, or the vacuity, according to the purport of the Question.

29.)

```
                 '  "  '''
29. ) 12—0 (0-4-11-7   Quote to be sought in the line of
——    ———              inches, against which in the line
2—5   2—4—0            of segments you will find
     ————————           '   "   '''
        1—5—0          4—7—8, by which if you mul-
     ————————          tiply the whole content, you
          ·—1          will have the gallons remaining.
```

```
137 — 3 — 8 whole content
  0 — 4 — 7 — 8
———————————————
 45 — 9 — 2 — 8
  5 — 8 — 7 — 10                     Observe, 6 seconds are
  0 — 11 — 5 — 3 — 8                 the 1/8 of 4 primes,
        7 — 7 — 6 — 5 — 4            & 8 thirds are the
———————————————————————              1/9 of 6 seconds.
 53 — 0 — 11 — 4 — 1 — 4 Facit. remaining.
———————————————————————
```

Now 12 inches being wet 17 are dry, I demand the vacuity, or how many gallons are wanting?

```
29. ) 17 (0-7-0-4-11   Quote to be sought in the line
——    ——               of inches, against which you
2—5   17—0             will find in the line of segments
     ——————————         '    "    '''
         1—0—0         7 — 4 — 4.
     ——————————
          2—4—0
     ——————————
            1—5
```

```
gall.    '    ''
137 — 3 — 8
  0 — 7 — 4 — 4
-----------------
 68 — 7 — 10
 11 — 5 — 3 — 8
  3 — 9 — 9 — 2 — 8
      3 — 9 — 9 — 2 — 8
-------------------------
 84 — 2 — 8 — 7 — 10 — 8 Facit wanting
 53 — 0 — 11 — 4 — 01 — 4 remaining
-------------------------
137 — 3 — 8 — 0 — 0 — 0 whole content for the
                           Proof.
```

Propofition. 2.

Having a Cylinders liquid capacity, and bung diameter, together either with the vacuity, or quantity of liquor remaining, to find the bung Diameters dry or wet part?

For the wet part, divide the remaining liquor by the whole content.

For the dry part, divide the vacuity by the whole content.

And feek their refpective quotes in the line of fegments, and againft it in the line of inches, you will find a number, by which if you multiply the bung diameter, the product will be the dry or wet inches, according to the purport of the queftion. Let the remaining Liquor of the fame Cylinder be 53 - 0 - 11 - 4 - 1 - 4 (gall. ' '' ''' '''' '''''). I demand how many inches of the bung diameter are wet?

```
137-3-8) 53-0-11-4-1-4 (0-4-7-8 Quote to be fought in
                                the line of fegments,
11-5-3-8  7-3-8-8-1             againft which in the
                                line of inches you will
              7-7-6-5-4         find 4 prim. 11 feconds
                                and 7 thirds.
```

0 — 4

	′	″	‴	
0	4	11	7	
			29	
2	10	9	1	Multiplyed by the ratio's of 29.
11	7	0	4	
	4	11	7	

Facit, 11—11—11—11 Are wet, which wants of 12 inches, but 1‴, which 1‴ remained on the first division, *viz.* 12 by 29.

Secondly, *Of a Cask taken as Spheroidal*, and as lying.

Here are five things to be known.

Bung Diameter	30 inches.
Head Diameter	25
Length	50
Wet	09

	gall.	′	″
Whole content	112	1	7

30.) 9 — 0 (0—3—7—2 Segment inches of bung diameter, to be sought in the line of inches. Against which in the line of segments stands 2—11—9.

2—6 1—6—0

·—6—0

```
gall.     '      ''
112 —— 1 —— 07
  0 —— 2 —— 11 —— 9
---------------------------------
 18 —— 8 —— 3 —— 2
  6 —— 2 —— 9 —— 0 —— 8
  1 —— 6 —— 8 —— 3 —— 2
       9 —— 4 —— 1 —— 7
       4 —— 8 —— 0 —— 9 —— 6
       2 —— 4 —— 0 —— 4 —— 9
-----------------------------------------
 27 —— 10 —— 0 —— 8 —— 7 —— 3 Facit
```

From whence if you take one gallon, it being the leſſer ſegment, there will remain 26, *&c.* which if you try, you will find it anſwers as near as any other way yet extant.

Now 9 inches being wet, 21 are dry, I demand the vacuity.

```
30. ) 21 —— 0 (0—8—4—9
----  ---------
2–6   1 —— 0 —— 0
      -------------
         2 —— 0 —— 0
         -------------
            1 —— 6 —— 0
```

Quote to be ſought in the line of inches, againſt which ſtands in the line of ſegments, 9 primes, 0 ſeconds 3 thirds.

```
gall.   '    ''
112 —— 1 —— 7
  0 —— 9 —— 0 —— 3
-------------------------
 56 —— 0 —— 9 —— 6
 28 —— 0 —— 4 —— 9
       2 —— 4 —— 0 —— 4 —— 9
---------------------------------
```

Facit 84 —— 3 —— 6 —— 3 —— 4 —— 9. To which if you add one gallon, it being the greater ſegment you will find it near enough the truth.

As

As for Parabolical Casks you may add and ſubtract, 1 gallon 3 primes, and you will find it near enough the truth alſo: and for Conical Casks you muſt add and ſubtract more, and how much more, a little practice will ſoon inform you. And to find the wet or dry inches of either bung diameter, you are to obſerve the ſame Rule with that for a Cylinder.

Now to find the contents of the ſegment of a Cask as ſtanding, take theſe following Rules.

First, *For a Spheroidal Cask.*

If the liquor be above the Casks mediety, find by the ſcale $\frac{2}{3}$ Area of the bung, and $\frac{1}{3}$ Area of the liquors ſuperficies, and multiply the ſum by the inches of the liquors depth from the bung, and add the ſemi-capacity thereunto, and you will have the ſegments content.

If the liquor be below the mediety add $\frac{2}{3}$ of the Area of the liquors ſuperficies, and $\frac{1}{3}$ of the Area of the head, and multiply their ſum by the depth, and you will have the ſegments content.

To find the dry or wet inches, divide the difference between the ſemi-content and vacuity, or remaining liquor by the Area of the mean circle, the quotient will be the diſtance from the bung-diameter to the ſurface of the liquor, which ſubtracted from or added to the ſemi-length, gives the dry or wet inches.

Secondly, *For the Fruſtum of a parabolical Spindle.*

To find the mean Area of the ſegment as ſtanding. If the liquor be above the mediety, take $\frac{1}{3}$ of the bung, and $\frac{1}{3}$ of the liquors ſuperficies; if below the mediety, take a third of the liquors ſuperficies, and a third of the head, (found by the ſcale,) in either caſe, to the ſum add half of the ſame, more the $\frac{1}{10}$ of the difference of Areas between bung and liquors

ſur-

ſurface, or head and liquors ſurface, (as the queſtion requires) (found by the Circle Gauge) the ſum will be the mean Area, which multiplyed by the depth of liquor from the bung, or from the head (as the queſtion requires) the product will be the content of the ſegment.

Thirdly, *For the Fruſtum of a Parabolical Conoid.*

To find the mean Area: To a third of the Area of the bung; if the ſurface of the liquor be above the mediety, but of the head if below the mediety, add a third of the Area of the liquors ſuperficies, and to their ſum add half thereof, and that ſum will be the mean Area, which multiplyed as before, gives the content of the ſegment.

Fourthly, *For the Conical Cask.*

To a third of the Area of the bung, if the ſurface of the liquor be above, but of the head, if below the mediety, add a third of the liquors ſuperficies, and to their ſum add the ſemi-ſum thereof, and from that ſum ſubduct the ſixth of the Area of the differing inches between bung and ſurface, or head and ſurface (as the queſtion requires) (found by the Circle Gauge) and the remain will be the mean Area, which multiplyed as before, gives the ſegments content.

To find the wet or dry inches of eithers bung diameter, you are to obſerve the ſame Rule before given.

POST-

POST-SCRIPT.

TO shew how useful this Arithmetick may be in other concerns, I shall give the Merchant a taste, to let him see how short and easie the French and Dutch Exchange may be wrought by it.

989 French Crowns, Exchange at 54 d. $\frac{3}{4}$ per Crown, to be received in Sterling money.

989
4—6—9

3956
494—6
61—9—9

4511|2—3—9

l. s. d.
225—12—3 $\frac{3}{4}$ *Facit.*

225 l. 12 s. 3 d. $\frac{3}{4}$ Sterling to be paid in French Crowns, at 54 d. $\frac{3}{4}$ the Exchange per Crown.

l. s. d.
225—12—3 $\frac{3}{4}$
20

4—6—9) 4512 (989 W *Facit.*

45—0—0

3—11—3

40—4—6

3—10—6

41—0—9

389 l.

389 l. Sterling to be paid in Flemiſh mony, Exchange at 33 s. 7 d. Flemiſh per l. ſterling.

389 l.
129 — 13 — 4
329 — 13 — 4
4 — 17 — 3

Flemiſh 653 — 03 — 11

653 l.03 s.11 d.Flemiſh to be paid in ſterling money, Exchange at 33 s. 7 d. Flemiſh per pound ſterling.

1 — 13 — 7) 653 — 3 — 11 (389 l. ſterling *Facit.*

6 — 0 — 0

0 — 19 — 3

14 — 12 — 6

1 — 3 — 10

15 — 2 — 3

· — · — ·

I ſhall add one queſtion more with its converſe, and ſo conclude.

A Wedge of Gold weighing 12 l. Averdupois, at 3 l.10 s. per ounce, I demand its value?

12

96

672 l. *Facit.*

What

What may a wedge of Gold weigh Averdupois, whose value amounts to 672 l. at 3 l. 10 s. per ounce.

672
—————
96
—————
12 l. *Facit.*

A Wedge of Gold weighing 12 l. Averdupois, valued worth 672 l. what is it worth per ounce?

672
—————
168
—————
42
—————
3 l. 10 s. *Facit.*

These Three last Questions, are wrought by the Contractions in Multiplication and Division.

FINIS.